MGF 1106. Miami Dade College

Chapter 3: Logic

- Logic, the science that studies the principles of correct reasoning.
- Statement or a proposition is a declarative sentence that is either true or false (but not both). A variable that represents a proposition is called a propositional variable, denoted p, q, or r.
- Negation: negation of statement p is *~p.* Also, observe that: *~ (~ p) = p.* *Example: It is not impossible = it is possible.*
- The negation of "All *A* are *B*" is "Some *A* are not *B*". In short: The negation of all is some and the negation of some is all.

  Negation examples:

  a) His calculator is not pink. Negation: His calculator is pink.
  b) Some students dislike math. Negation: All students loves math.
  c) All students loves math. Negation: Some students dislike math.
  d) No cat flies. Negation: Some cats fly. Or, at least one cat flies.

- Statements that involve one or more of the *connectives* ``and'', ``or'', ``not'', ``if . . . then '' and `` . . . if and only if . . . '' are *compound* statements. Example: It will rain today and tomorrow. It will rain or tomorrow. If it rain today then It rain tomorrow.
- If p and q are statements, then, in symbolic form: negation (*~p)*, conjunction ($p \wedge q$) , disjunction($p \vee q$), *conditional* ($p \rightarrow q$) *and the* Biconditional ($p \leftrightarrow q$).

- Basic Truth Tables:

Negation

p	~ p
t	F
F	T

Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Conditional

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional

p	q	$p \leftrightarrow q$
T	T	T
t	F	F
F	T	F
F	F	T

- **Equivalent compound statements** are made up of the same simple statements and have the same corresponding truth values for all true-false combinations of these simple statements. If a compound statement is true, then its equivalent statement must also be true.

  Variations of the Conditional Statement: $p \rightarrow q$

  Inverse: negate the components: ~p → ~q

  Converse: reverse the components: q → p

  Contrapositive: the inverse of the converse: ~q → ~p

  Equivalences: $p \rightarrow q$ *equivalent to* ~p ∨ q

  $p \rightarrow q$ *equivalent to* ~q→ ~p

Examples:

If a shape has three sides, then the shape is a triangle. (Conditional)
If a shape does not have three sides, then the shape is not a triangle. (Inverse)
If a shape is a triangle, then the shape has three sides. (Converse)
If a shape is not a triangle, then it does not have three sides. (Contrapositive)
A shape is a triangle if and only if the shape has three sides. (Biconditional)

- Negations of Conditional Statements: ~ $(p \rightarrow q)$ is p $\wedge$ ~q.
- **De Morgan's Laws:** First law: ~ **(**p $\wedge$ $q)$ = ~ p $\vee$ ~ q
  The negation of *p and q* *is not p or not q.*
- Second law: ~ **(**p $\vee$ $q)$ = ~ p $\wedge$ ~ q
  The negation of *p or is Not p and not p.*

Using De Morgan's Laws, show that the negation of *if p then q, is p and not q.*
Answer: In symbols ~ $(p \rightarrow q)$ is p $\wedge$ ~q. Considering that $p \rightarrow q$ *is* equivalent to ~ p $\vee$ q; then,
~ $(p \rightarrow q)$ = ~(~p $\vee$ q)= ~~p $\wedge$~ q= p $\wedge$ ~ q.

Arguments:

Valid Argument: The conclusion is true whenever the premises are assumed to be true.

Invalid Argument: Also called a fallacy

Testing the Validity of an Argument with a Truth Table: [(premise 1) $\wedge$(premise 2) $\wedge$ (premise n)] $\rightarrow$conclusion.
If the final column is all T, then the statement is a tautology and the argument is valid.

Example of argument:

If Mr. Scott is still with us, then the power will come on.
The power comes on.
Therefore, Mr. Scott is still with us.

Determine whether the argument is valid or invalid.

Solution:

Step 1: *p*: Mr. Scott is still with us.

q: The power will come back on.

Step 2: Write the argument in symbolic form:

$p \rightarrow q$ If Mr. Scott is still with us, then the power will come on.

$\underline{q}$ The power comes on.

$\therefore$ p Mr. Scott is still with us.

Step 3 : Write the symbolic statement.

$$[(p \rightarrow q) \wedge q] \rightarrow p$$

Construct a truth table.

p	q	$p \rightarrow$ q	$(p \rightarrow$ q) ^ q	$[(p \rightarrow$ q) ^ $q] \rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Conclusion: the entries in the final column are not all true, so the conditional statement is not a tautology. Spock's argument is invalid or a fallacy.

Standard Forms of Arguments:

Valid Arguments			
Direct Reasoning	**Contrapositive Reasoning**	**Disjunctive Reasoning**	**Transitive Reasoning**
$p \to q$ $\underline{p}$ $\therefore q$	$p \to q$ $\underline{\sim q}$ $\therefore \sim p$	$p \vee q$ $\quad$ $p \vee q$ $\underline{\sim p}$ $\quad$ $\underline{\sim q}$ $\therefore q$ $\quad$ $\therefore p$	$p \to q$ $\underline{q \to r}$ $\therefore p \to r$ $\therefore \sim r \to \sim p$
Invalid Arguments			
Fallacy of the Converse	**Fallacy of the Inverse**	**Misuse of Disjunctive Reasoning**	**Misuse of Transitive Reasoning**
$p \to q$ $\underline{q}$ $\therefore p$	$p \to q$ $\underline{\sim p}$ $\therefore \sim q$	$p \vee q$ $\quad$ $p \vee q$ $\underline{p}$ $\quad$ $\underline{q}$ $\therefore \sim q$ $\quad$ $\therefore \sim p$	$p \to q$ $\underline{q \to r}$ $\therefore r \to p$ $\therefore \sim p \to \sim r$

Example of **Disjunctive syllogism**:

Either the Sun orbits the Earth, or the Earth orbits the Sun.
The Sun does not orbit the Earth.
Therefore, the Earth orbits the Sun.

Exercises:

1. Construct a truth table for $\sim(p \wedge q)$
2. Construct a truth table for $(\sim p \vee q) \wedge \sim q$.
3. Construct a truth table for $[(p \vee q) \wedge \sim p] \to q$
4. Prove the first De Morgan's law using a truth table: $\sim (p \wedge q) = \sim p \vee \sim q$

Set up for #1:

p	q	$p \wedge q$	$\sim(p \wedge q)$

Set up for #2:

p	q	$\sim p$	$(\sim p \vee q)$	$\sim q$	$(\sim p \vee q) \wedge \sim q$

Set up for #3:

p	q	$(p \vee q)$	$\sim p$	$[(p \vee q) \wedge \sim p]$	$[(p \vee q) \wedge \sim p] \to q$

Set up for #4:

p	q	$p \wedge q$	$\sim (p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$

For this column and this column corresponding truth-values coincide.