Confidence Intervals:

A point estimate is a specific numerical value estimate of a parameter. The best point estimate of the population mean μ is the sample mean $\bar{x}$.

An interval estimate of a parameter is an interval or a range of values used to estimate the parameter. This estimate may or may not contain the value of the parameter being estimated.

The confidence level of an interval estimate of a parameter is the probability that the interval estimate will contain the parameter, assuming that many samples are selected and that the estimation process on the same parameter is repeated.

A confidence interval is a specific interval estimate of a parameter determined by using data obtained from a sample and by using the specific confidence level of the estimate.

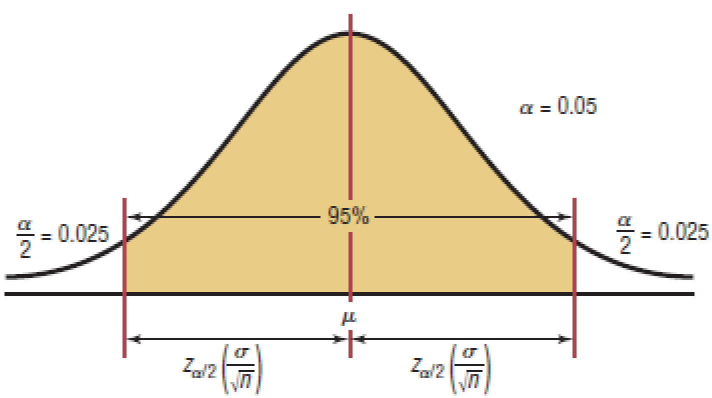
Intervals constructed in this way are called confidence intervals. Three common confidence intervals are used: the 90, the 95, and the 99% confidence intervals.

The central limit theorem states that when the sample size is large, approximately 95% of the sample means taken from a population and same sample size will fall within $\pm$ 1.96 standard errors of the population mean, that is,

$\mu \pm 1.96\left(\frac{\sigma}{\sqrt{n}}\right)$ for a 95% CI, in general: $\mu \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$ for all $z_{\alpha/2}$

For a 90% confidence interval, $z_{\alpha/2} = 1.65$; for a 95% confidence interval, $z_{\alpha/2} = 1.96$; and for a 99% confidence interval, $z_{\alpha/2} = 2.58$.

The term $z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$ is called the maximum error of the estimate (also called the margin of error).



Formula for the Minimum Sample Size Needed for an Interval Estimate of the Population Mean:

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

Confidence Intervals for the mean when σ is unknown:

When σ is known and the sample size is 30 or more, or the population is normally distributed if sample size is less than 30, the confidence interval for the mean can be found by using the z distribution.

However, most of the time, the value of σ is not known, so it must be estimated by using s, namely, the standard deviation of the sample. When s is used, especially when the sample size is small, critical values greater than the values for $z_{\alpha/2}$ are used in confidence intervals in order to keep the interval at a given level. These values are taken from the Student t distribution, most often called the t distribution.

The t distribution differs from the standard normal distribution in the following ways:

1. The variance is greater than 1.

2. The t distribution is a family of curves based on the concept of degrees of freedom, which is related to sample size.

3. As the sample size increases, the t distribution approaches the standard normal distribution.

Intervals are calculated as follows:

$$\bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

Confidence Intervals and Sample Size for Proportions:

A proportion represents a part of a whole. It can be expressed as a fraction, decimal, or percentage.

As with means, the statistician, given the sample proportion, tries to estimate the population proportion. Point and interval estimates for a population proportion can be made by using the sample proportion. For a point estimate of p (the population proportion), $\hat{p}$ (the sample proportion) is used.

The confidence interval for a given p is based on the sampling distribution of $\hat{p}$. When the sample size n is no more than 5% of the population size, the sampling distribution of $\hat{p}$ is approximately normal with a mean of p and a standard deviation of $\sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$ where $\hat{q} = 1 - \hat{p}$.

Therefore, the population proportion interval is calculated as follows:

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$$

Sample Size for Proportions

To find the sample size needed to determine a confidence interval about a proportion, use this formula:

$$\mathrm{n} = \hat{p} \cdot \hat{q} \left(\frac{z_{\alpha/2}}{E}\right)^2$$

If necessary, round up to obtain a whole number.