

## Hypothesis Testing Notes

A statistical hypothesis is a conjecture about a population parameter. This conjecture may or may not be true.

There are two types of statistical hypotheses for each situation: the null hypothesis and the alternative hypothesis.

These hypotheses contain opposing viewpoints.

The null hypothesis, symbolized by  $H_0$ , is a statistical hypothesis that states that there is no difference between a parameter and a specific value, or that there is no difference between two parameters.

The alternative hypothesis, symbolized by  $H_1$  or  $H_a$ , is a statistical hypothesis that states the existence of a difference between a parameter and a specific value, or states that there is a difference between two parameters.

Since the null and alternative hypotheses are contradictory, we must examine evidence to decide if you have enough evidence to reject the null hypothesis or not. The evidence is in the form of sample data.

Note:  $H_0$  always has a symbol with an equal in it.  $H_a$  never has a symbol with an equal in it. The choice of symbol depends on the wording of the hypothesis test.

Examples:

#1

No more than 30% of the registered voters in Santa Clara County voted in the primary election.

More than 30% of the registered voters in Santa Clara County voted in the primary election.

**In symbols:**

or

$$H_0: p \leq 0.30$$

$$H_a: p > 0.30$$

$$H_0: p = 0.30$$

$$H_a: p > 0.30$$

#2

We want to test whether the mean GPA of students in American colleges is different from 2.0 (out of 4.0).

The null and alternative hypotheses are:

$$H_0: \mu = 2.0$$

$$H_a: \mu \neq 2.0$$

**Rare Events:**

Suppose you make an assumption about a property of the population (this assumption is the null hypothesis). Then you gather sample data randomly. If the sample has properties that would be very unlikely to occur if the assumption is true, then you would conclude that your assumption about the population is probably incorrect.

A statistical test uses the data obtained from a sample to make a decision about whether the null hypothesis should be rejected. The numerical value obtained from a statistical test is called the test value.

One sample Hypothesis tests ---**Formulas and calculations:**

1. One mean, sigma known:

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

2. One mean, sigma unknown:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

3. One proportion:

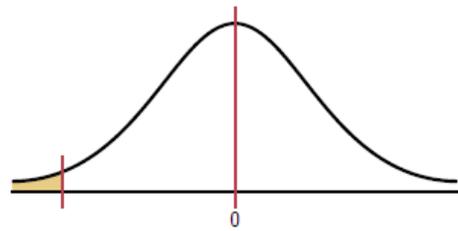
$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}$$

**Left tail, right tail, two tails tests** depend on the symbol used in the alternative hypothesis: <, >, ≠ respectively

**Summary of Hypothesis Testing and Critical Values**

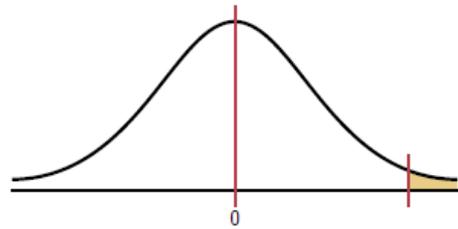
$$\begin{matrix} H_0: \mu = k \\ H_1: \mu < k \end{matrix} \begin{cases} \alpha = 0.10, \text{ C.V.} = -1.28 \\ \alpha = 0.05, \text{ C.V.} = -1.65 \\ \alpha = 0.01, \text{ C.V.} = -2.33 \end{cases}$$

**(a) Left-tailed**



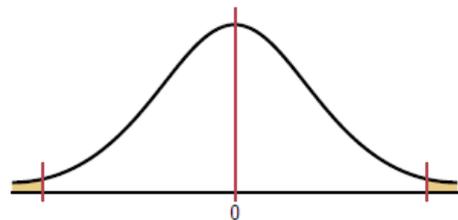
$$\begin{matrix} H_0: \mu = k \\ H_1: \mu > k \end{matrix} \begin{cases} \alpha = 0.10, \text{ C.V.} = +1.28 \\ \alpha = 0.05, \text{ C.V.} = +1.65 \\ \alpha = 0.01, \text{ C.V.} = +2.33 \end{cases}$$

**(b) Right-tailed**



$$\begin{matrix} H_0: \mu = k \\ H_1: \mu \neq k \end{matrix} \begin{cases} \alpha = 0.10, \text{ C.V.} = \pm 1.65 \\ \alpha = 0.05, \text{ C.V.} = \pm 1.96 \\ \alpha = 0.01, \text{ C.V.} = \pm 2.58 \end{cases}$$

**(c) Two-tailed**



### Solving Hypothesis-Testing Problems (**Traditional Method**)

Step 1: State the hypotheses and identify the claim.

Step 2: Find the critical value(s) from the appropriate table.

Step 3: Compute the test value.

Step 4: Make the decision to reject or not reject the null hypothesis.

Step 5: Summarize the results.

The P-value (or probability value) is the probability of getting a sample statistic (such as the mean) or a more extreme sample statistic in the direction of the alternative hypothesis when the null hypothesis is true.

### Solving Hypothesis-Testing Problems (**P-Value Method**)

Step 1: State the hypotheses and identify the claim.

Step 2: Compute the test value.

Step 3: Find the P-value.

Step 4: Make the decision.

Step 5: Summarize the results.

Calculating p-values:

a. If  $z$  (test statistics) is negative, that is a left tailed test, p-value is the area (probability) to the left of  $z$ , given by tables. For a two tailed test, multiply that area by two.

b. If  $z$  (test statistics) is positive that is a right tailed test, p-value is the area (probability) to the right of  $z$ . And, area to the right is  $= 1 - \text{area on table (left)}$ . For a two tailed test, multiply that area by two.

On graphing calculators, hypothesis tests procedures yield p-values. Otherwise, use DIST, Normal cdf on the calculator.

If  $P\text{-value} \leq \alpha$ , reject the null hypothesis.

If  $P\text{-value} > \alpha$ , do not reject the null hypothesis.

### **Errors:**

In the hypothesis-testing situation, there are four possible outcomes.

1. Reject the Null Hypothesis when the null is in fact false. That is a correct decision.
2. Reject the Null Hypothesis when it is true. That is a mistake. It is called a **type I** error.
3. Fail to reject the Null Hypothesis when it in fact it is true, that is a correct decision.
4. Fail to reject the Null Hypothesis when it is false, that is a mistake: It is called a **type II** error.

**Example 1:** 1,500 women followed the Atkin's diet for a month. A random sample of 29 women gained an average of 6.7 pounds. Use a 0.05 level of significance to test the claim that the average weight gain per woman for the month was over 5 pounds. The standard deviation for all women in the group was 7.1

$H_0: \mu = 5$

$H_1: \mu > 5$

Significance level,  $\alpha = 0.05$ ; right tailed test. The  $z_{0.05} = 1.645$ , which is the critical value.

Since the problem states that the standard deviation of all women was 7.1; that is, sigma ( $\sigma$ ) is given, so we use a Z-Test.

Test Statistic:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{6.7 - 5}{\frac{7.1}{\sqrt{29}}} = 1.29$$

Calculator: Z-Test on TI 83 & 84:

```
Z-Test
Inpt:Data Stats
μ₀:5
σ:7.1
x̄:6.7
n:29
μ:≠μ₀ <μ₀ >μ₀
Calculate Draw
```

```
Z-Test
μ>5
z=1.289405658
P=.098628613
x̄=6.7
n=29
```

On Casio 9750:

```
1-Sample ZTest
Data Variable
μ₀      :>μ₀
μ₀      :5
σ       :7.1
x̄       :6.7
n       :29
List Var ↓
```

```
1-Sample ZTest
μ      >5
z      =1.28940566
P      =0.09862854
x̄      =6.7
n      =29
```

Conclusions: Fail to reject the Null hypothesis. According to the sample data there is not enough evidence to support the claim that the average weight gain per woman was over five pounds.

**Example 2.** The mean temperature in the town of ALPHA for the month of January is 60 degrees Fahrenheit. A sample of 20 days is taken, the mean temp for those 20 days is 57 degrees, the standard deviation is 3 degrees. Use a 0.01 level of significance to test the claim.

$$H_0: \mu = 60$$

$$H_1: \mu \neq 60$$

Note: the claim states a value; Temp (..) is equal to 60; it does not say greater or less, just a value. In this case the alternative is not equal to ( $\neq$ ).

The sample mean and standard deviations are given; therefore, this is a T-Test.

Level of significance is 0.01 ( $\alpha = 0.01$ ); because the alternative hypothesis uses  $\neq$ , the test is a two tailed test. The  $t_{\alpha/2}$  critical value, for  $20 - 1 = 19$  degrees of freedom by the T-Table is  $t_{0.005} = 2.861$ .

Remember, the critical value is needed in order to establish our conclusions, when we don't use a graphing calculator: we compare the T-test statistic, which we are calculating below, to the critical value: when the absolute value of the test-test is greater than the critical value we Reject the Null hypothesis. Let's find the test stat:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{57 - 60}{\frac{3}{\sqrt{20}}} = -4.47$$

Calculators, TI 83 & 84:

```
T-Test
Inpt: Data Stats
μ₀: 60
x̄: 57
Sx: 3
n: 20
μ: ≠μ₀ <μ₀ >μ₀
Calculate Draw
```

```
T-Test
μ≠60
t=-4.472135955
P=2.6119338E-4
x̄=57
Sx=3
n=20
```

Casio 9750:

```
1-Sample tTest
Data : Variable
μ₀ : 60
x̄ : 57
sx : 3
n : 20
List Var ↓
```

```
1-Sample tTest
μ : ≠60
t : -4.472136
P : =2.6119E-04
x̄ : =57
sx : =3
n : =20
```

The output of the t-test shows a t-statistic,  $t = -4.47$  and a p-value =  $2.6 \times 10^{-4} = 0.00026$

Conclusions: Reject the Null Hypothesis. There are two ways of drawing this conclusion: the t-stat absolute value is greater than the critical value; or the p-value is less than the stated value of alpha. In technical terms: we have enough sample evidence to warrant rejection of the claim that the mean temperature is 60 degrees Fahrenheit

**Example 3:** The mayor of the town of ALPHA claims that over 60% of the citizens approve his job as mayor. A random sample of 200 residents, all voters of the town, are interviewed; 130 of them agrees with the mayor. Use a 0.05 level of significance to test the claim.

$H_0: p = 0.60$

$H_1: p > 0.60$

One proportion Z-Test.  $\alpha = 0.05$ ; right tailed test.  $Z_{0.05} = 1.645$

First, calculate the sample proportion:

$$\hat{p} = \frac{x}{n} = \frac{130}{200} = 0.65$$

Z- test Statistic:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.65 - 0.60}{\sqrt{\frac{0.60 * 0.40}{200}}} = 1.44$$

Calculators, TI 83 & 84:

```
1-PropZTest
P0: .6
x: 130
n: 200
PROP#P0 <P0 >P0
Calculate Draw

1-PropZTest
PROP>.6
z=1.443375673
P=.074457379
P̂=.65
n=200
```

Casio 9750:

```
1-Prop ZTest
PROP :>P0
P0 : 0.6
x : 130
n : 200
Save Res:None
Execute
|CALC

1-Prop ZTest
PROP>0.6
z =1.44337567
P =0.07445733
P̂ =0.65
n =200
```

Calculator: test- statistic  $z = 1.44$ ; p-value = 0.074

Conclusions: We fail to reject the Null hypothesis. In technical terms: there is not enough evidence to support the claim that over 60% of the citizens approve the mayor's job.