

Two Samples Hypothesis Testing

Testing the Difference Between Two means, independent samples:

Sometimes a researcher needs to determine whether the means of the two different groups are equal. The hypothesis may be expressed as follows:

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_1: \mu_1 - \mu_2 \neq 0$$

The Null hypothesis establishes that there is no difference between the two means; the alternative, in this previous case, that the means are different. There are two other cases: the mean of group 1 is greater than the mean for group 2, as follows:

$$H_1: \mu_1 - \mu_2 > 0$$

Or, mean in group 1 less than mean in group 2:

$$H_1: \mu_1 - \mu_2 < 0$$

Assumptions for the Test to Determine the Difference Between Two Means:

1. The samples must be independent of each other. That is, there can be no relationship between the subjects in each sample.
2. The standard deviations of both populations must be known, and if the sample sizes are less than 30, the populations must be normally or approximately normally distributed.

Formula for the z Test for Comparing Two Means from Independent Populations

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

On TI 83 – 84:

STAT → TESTS → 2-SamZTest:

```
EDIT CALC TESTS
1:Z-Test...
2:T-Test...
3:2-SampZTest...
4:2-SampTTest...
5:1-PropZTest...
6:2-PropZTest...
7↓ZInterval...
```

Hit enter:

```
2-SampZTest
Inpt:Data Stats
σ1:0
σ2:0
x̄1:0
n1:0
x̄2:0
↓n2:0
```

Input values, press enter.

It yields test statistics z and p, the p-value.

For example:

```
2-SampZTest
μ1≠μ2
z=-2.773500981
P=.005545783
x̄1=21
x̄2=23
↓n1=25
```

On Casio 9750:

In STAT mode, press F3 for TEST, then F1 for Z, then F2 for 2-S (Two samples):

```
2-Sample ZTest
Data : Variable
μ1    : >μ2
σ1    : 7.1
σ2    : 1
x̄1    : 57
n1    : 200      ↓
List Var
```

it yields:

```
2-Sample ZTest
μ1    : >μ2
z     : -2.5642217
P     : 0.99482962
x̄1    : 59.7
x̄2    : 61
n1    : 200
```

Testing the Difference Between Two Means: Dependent Samples:

Samples are considered to be dependent samples when the subjects are paired or matched in some way.

When the samples are dependent, a special t test for dependent means is used. This test employs the difference in values of the matched pairs. The hypotheses are as follows:

Two-tailed	Left-tailed	Right-tailed
$H_0: \mu_D = 0$ $H_1: \mu_D \neq 0$	$H_0: \mu_D = 0$ $H_1: \mu_D < 0$	$H_0: \mu_D = 0$ $H_1: \mu_D > 0$

Where μ_D is the mean of the differences between the two samples.

Test value, using the formula:

$$t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} \quad \text{degrees of freedom } df = n - 1$$

When using a hypothesis test for matched or paired samples, the following characteristics should be present:

1. Simple random sampling is used.
2. Sample sizes are often small.
3. Two measurements (samples) are drawn from the same pair of individuals or objects.
4. Differences are calculated from the matched or paired samples.
5. The differences form the sample that is used for the hypothesis test.
6. Either the matched pairs have differences that come from a population that is normal or the number of differences is sufficiently large so that distribution of the sample mean of differences is approximately normal.

The t test for the mean of the differences we simply perform a t – test the same way we proceed for one sample since after finding the differences between the two means we only have a set a values (the differences).

In TI 83 or 84: STAT → TESTS → T-Test.

In Casio 9750: in STAT mode F3 for TEST, then F2 for t then 1-S (One sample).

Example 1:

Arianna Estefan manages Salmon Falls Park, which has two flagship roller coasters, the Flyer and the Destroyer. When Arianna walks around the park during hours of operation, she notices that there are fewer people waiting in line for the Flyer than for the Destroyer. Since both roller coasters have similar capacities and similar run times, both roller coasters should have about the same number of riders. Based on the anecdotal evidence, Arianna claims that the population mean number of riders per hour on the Flyer is less than the population mean number of riders per hour on the Destroyer. Arianna reviews data collected in the past and assumes that the population standard deviation is 48.29 for the Flyer and 51.86 for the Destroyer. Arianna randomly selects hours during which the attendant of each ride counts the number of riders. The sampling occurs over the course of several weeks. The results are provided in the table shown below. Let μ_1 be the population mean number of riders per hour on the Flyer and μ_2 be the population mean number of riders per hour on the Destroyer. Using $\alpha = 0.05$ test Arianna's claim.

Flyer: $\bar{x} = 109.62$ $\sigma_1 = 48.29$ $n_1 = 54$

Destroyer: $\bar{x} = 124.71$ $\sigma_2 = 51.86$ $n_2 = 51$

Answer:

This is a left-tailed test because the alternative hypothesis is:

$$H_a: \mu_1 - \mu_2 < 0$$

Critical value: $z = -1.645$

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{109.62 - 124.71}{\sqrt{\frac{48.29^2}{54} + \frac{51.86^2}{51}}} = -1.54$$

Conclusion: Since Z_0 is not in the rejection area; we fail to reject the Null hypothesis of the equality of the two means. There is not enough evidence to support Arianna's claim that the population mean number of riders per hour on the Flyer is less than the population mean number of riders per hour on the Destroyer.

Find t p-values on tables (and Calculators):

Question

Your friend believes that he has found a route to work that would make your commute faster than what it currently is under similar conditions. Suppose that data were collected for a random set of 7 days, where each difference is calculated by subtracting the time taken on the current route from the time taken on the new route. Assume that the populations are normally distributed. Your friend uses the alternative hypothesis $H_a : \mu_d < 0$. Suppose the test statistic t is computed as $t \approx -3.201$, which has 6 degrees of freedom. What range contains the p-value?

Probability	0.10	0.05	0.025	0.01	0.005
Degrees of Freedom					
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947

From table $0.005 < p < 0.01$

Ans: $p < 0.01$

On Calc t cdf from lower -E99, upper -3.201, df = 6 yields a p-value $p = 0.00928$ which is indeed greater than 0.005 but less than 0.01

Notice that this a one tailed test; otherwise we need to multiply the calculated value by 2 (for two tailed tests).

Question

A physician wants to determine if a supplement is effective in helping men lose weight. She takes a random sample of overweight men and records their weights before the trial. She then prescribes the supplement and instructs them to take it for four weeks while making no other lifestyle changes. After the four-week period, she records the weight of the men again. Suppose that data were collected for a random sample of 6 men, where each difference is calculated by subtracting the weight before the trial from the weight after the trial. Assume that the populations are normally distributed. The physician uses the alternative hypothesis $H_a : \mu_d < 0$. Using a test statistic of $t \approx -2.795$, which has 5 degrees of freedom, determine the range that contains the p-value.

Probability	0.10	0.05	0.025	0.01	0.005
Degrees of Freedom					
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947

That is: $0.025 > p > 0.01$ better: $0.01 < p < 0.025$

Answer:

$0.01 < p < 0.05$

On Calc t cdf from lower -E99, upper -2.795 df = 5 yields p-value $p = 0.0191$ which is indeed greater than 0.01 but less than 0.05

Question

A researcher wants to test to see if husbands are significantly older than their wives. To do this, he collects the ages of husbands and pairs them with the ages of their respective wives for a random set of married couples. Suppose that data were collected for a random sample of **12** couples, where each difference is calculated by subtracting the age of the wife from the age of the husband. Assume that the ages are normally distributed. The researcher uses the alternative hypothesis $H_a : \mu_d > 0$. Using a test statistic of $t \approx 1.434$, which has **11** degrees of freedom, determine the range that contains the p-value.

Probability	0.10	0.05	0.025	0.01	0.005
Degrees of Freedom					
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947

Ans: $0.10 > p > 0.05$ better $0.05 < p < 0.10$

On Calc t cdf from lower 1.434, upper E99 df = 11 yields p-value $p = 0.089$ which is greater than 0.05 but less than 0.10

Question

A real estate agent believes that the value of houses in the neighborhood she works in has increased from last year. To test this claim, she selects random houses in this neighborhood and compares their estimated market value in the current year to their estimated market value in the previous year. Suppose that data were collected for a random sample of 8 houses, where each difference is calculated by subtracting the market value of the previous year from the market value of the current year. Assume that the values are normally distributed. The agent uses the alternative hypothesis $H_a : \mu_d > 0$. Using a test statistic of $t \approx 7.496$, which has 7 degrees of freedom, determine the range that contains the p-value.

Probability	0.10	0.05	0.025	0.01	0.005
Degrees of Freedom					
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947

$0.005 > p$ or better $p < 0.005$

Select the correct answer below:

- p-value > 0.10
- $0.05 < p\text{-value} < 0.10$
- $0.01 < p\text{-value} < 0.05$
- p-value < 0.01

On calculator: t cdf from lower 7.496, upper E99 df = 7 yields p-value $p = 0.000068..$ which is less than 0.005 and, therefore, less than 0.01