Hypothesis Testing

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Step by Step Process of Statistical Hypothesis Testing

1: State the Null Hypothesis:
A hyphotetized population parameter set equal to an specific value: it is assumed that there is no significant difference between specified populations. Symbols for one sample, proportions or means: $H_0 : p =$ or $H_0 : \mu =$

2: State the Alternative Hypothesis: The alternative hypothesis is the researcher hypothesis.
Symbols for one sample, proportions: $H_1 : p <$ or $H_1 : p \neq$ or $H_1 : p >$
Symbols for one sample, means: $H_1 : \mu <$ or $H_1 : \mu \neq$ or $H_1 : \mu >$

3: Set α, the level of the significance and establish the critical values. In the software era, p-values are used instead of critical values.

4: Collect the data: that is, the researcher collect data in order to test the null hypothesis. Of course, the objective of the researcher is finding data to contradict the null hypothesis. This is the sample data.

5: Calculate the test statistic: in this step we use the value of the population parameter stated on the Null and the sample data:

a) For one proportion:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

Where p is the population proportion set by the Null Hypothesis;$\hat{p}$ is the sample proportion: $\hat{p} = \frac{x}{n}$

b) For one mean, when the population standard deviation is known:

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

c) For one mean, when the population standard deviation is unknown:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

6: Draw a conclusion about H_0 based on the test statistic:
Choose on of two methods to draw your conclusion:

Method 1. The classical method based on critical values, that consists on finding the z_α or the $z_{\alpha/2}$ in the case of testing one proportion or testing a population mean when σ is known. Otherwise, for the $t - test$ –population standard deviation unknown– use the t_α or $t_{\alpha/2}$. Critical values are found in the Normal distribution table or the T-Distribution table of critical values. If the absolute value of the test statistic is greater than the critical value, then we reject the Null; otherwise, we fail to reject the Null.

Method 2. The p-value method. The p-value of the test is displayed by software of graphing calculators. The p-value is the probability of the observed data, given that the null hypothesis is true.
At this point the Rare Event Rule for Inferential Statistics applies: if under a given assumption, the probability of a particular observed event is extremely rare, we conclude that the assumption is probably incorrect.

In short, when the p-value is less than α we reject the Null; otherwise, we fail to reject the Null.

Note: For two samples the procedure is the same, except that the test statistic carries a different formula:

For two sample proportions: testing $H_0 : p_1 = p_2$:

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}.$$

For Two-Sample means t-statistic when σ_1 and σ_2 are unknown and not assumed to be equal:

$$t = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

If σ_1 and σ_2 are unknown and assumed to be equal, then we calculate the pooled standard deviation using the formula for s_p which replaced s_1 and s_2 in the formula for t stated above:

$$s_p^2 = \frac{(n_1 - 1) \cdot s_1^2 + (n_2 - 1) \cdot s_2^2}{n_1 + n_2 - 2}$$

Finding Test Statistic and p-values by Graphing Calculators

Note: instructions on imathesis.com: Hypothesis Testing by CASIO Graphing Calculators; and, Hypothesis Testing by TI Graphing Calculators.

TI83 and 84:

Press STAT, Choose TESTS, then, for proportions: 1-PropZTest; for Two Proportions, 2-PropZTest; For means, if σ known, Z-Test; otherwise, T-Test. For Two samples means, 2-SampTTest or 2-SampZTest.

Casio 9750GII or 9860GII:

From main STAT screen press F3 for TEST, press F1 and choose Z for Proportions or for means when σ is known; otherwise, choose F2, t. Once Z or t is selected, 1-S stand for one-sample mean, 2-S, two sample means and 1-P for one Proportion while 2-P for two proportions.