Practice 7

imathesis.com

By Carlos Sotuyo

Worked answers to Practice 7, Confidence intervals

1) $Z_{\alpha/2}$ for 98%:

Find $\alpha = 1 - 0.98 = 0.02$; then find the corresponding Z score for $1 - \alpha/2 = 1 - 0.02/2 = .99$; by the Normal distribution table, $Z = 2.33$

Using a graphing calculator, choose InvNorm, area $= 1 - \alpha/2$; keep mean, $\mu = 0$; $\sigma = 1$. It yields $2.3263 \approx 2.33$

2) Following same procedure: $\alpha = 1 - 0.91 = 0.09$; then, $1 - \alpha/2 = 1 - 0.09/2 = 0.955$; In the z score table the corresponding z is 1.70; by graphing calculator: InvNorm of 0.955 yields $1.69539 \approx 1.70$.

3) Again, find α: $\alpha = 1 - 0.96 = 0.04$; then, $1 - \alpha/2 = 1 - 0.04/2 = 0.0.98$; In the z score table the corresponding z is 1.70; by graphing calculator: InvNorm of 0.98 yields $2.0537 \approx 2.05$.

4) Let's identify the lower bound of the interval as L, for Left; and the upper bound as R, for Right. Then, $\hat{p} = \frac{R+L}{2} = \frac{0.579+(-0.061)}{2} = 0.259$; for the error, $E = \frac{R-L}{2} = \frac{0.579-(-0.061)}{2} = 0.32$.

Therefore, the answer is A.

5) Just find the sample proportion, $\hat{p} = \frac{R+L}{2} = \frac{0.822+(0.862)}{2} = 0.842$

6) Same procedure as #5: $\hat{p} = \frac{R+L}{2} = \frac{0.686+(0.712)}{2} = 0.699$

7) Using a graphing calculator you may find the interval, say for TI 83 or 84, STAT, TEST, 1-PropZint; update x, n and confidence level. It yields $(.07851, .13426)$, and then calculate the margin of Error using: $E = \frac{R-L}{2} = \frac{.13426 - 0.07851}{2} = 0.027875 \approx 0.0279$

Using the formula for margin for Error (one proportion: $E = Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{0.1064 * 0.8936}{470}} = 0.027877 \approx 0.0279$.

Recall that $\hat{p} = \frac{x}{n} = \frac{50}{470} = 0.1064$ and $\hat{q} = 1 - \hat{p} = 1 - 0.1064 = 0.8936$; and the $Z_{\alpha/2}$ for 95% is 1.96

8) Same type of question as #7; this time the $Z_{\alpha/2}$ for 90% is 1.645

9) For Questions from 9 to 12, if using a graphing calculator, follow the instructions on how to create Confidence Intervals for one proportion. On imathesis.com: Instructions on how to construct CIs using TI83 and 84; or, Instructions on how to construct CIs using Casio9750GII and 9860GII.

Using formulas: First, find $\hat{p} = \frac{x}{n}$; then, $\hat{q} = 1 - \hat{p}$; use the specific $Z_{\alpha/2}$ [for 95% = 1.96; for 98% = 2.33; for 88% = 1.55, for 90% = 1.645] and use the formula for Error: $E = Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$; once we known the Error, add the Error to $\hat{p}$ that yields the right side or upper bound of the interval; and subtract the Error from $\hat{p}$, in order to obtain the lower bound or left side of the interval. This procedure is described by the formula $\hat{p} \pm E$.

13) From 13 to 16, we need to find minimum sample size for proportions. Use the formula:

$$n = \frac{[Z_{\alpha/2}]^2 \cdot \hat{p} \cdot \hat{q}}{E^2}$$

Whenever $\hat{p}$ and $\hat{q}$ are unknown, we enter 0.50 for each of them; so, the product of the two values is always 0.25; again, whenever they are unknown; otherwise, enter the given values; as a rule only $\hat{p}$ is given, find $\hat{q} = 1 - \hat{p}$.

The $Z_{\alpha/2}$ for 99% = 2.575; for 95% = 1.96

Remember that the result of the calculation is always round up to the next integer.

17) and 18) It is about finding $\hat{p} = \frac{x}{n}$

In a proportion it is always part over total; part is x, total is n.

19) to 22) are similar to questions from 9) to 12) except that this time x, n are not explicitly stated. The confidence level is given, we may use a graphing calculator or the formula (see details in #9).

23) to 25). These questions are related to confidence intervals for sample mean. Since in these cases σ is not given, but s instead (the sample standard deviation), we construct T-Intervals. Follow the procedure in a graphing calculator or, use the formula:

First, find the Error: $E = t_{\alpha/2} \frac{s}{\sqrt{n}}$

In order to find $t_{\alpha/2}$ use the t table of critical values, look up degrees of freedom equal to one less than n. For example for #10 $df = 10 - 1 = 9$ and the the corresponding confidence level. Once the Error is calculated, add the error to the sample mean $\bar{x}$ which yields the upper bound or right side of the interval; and subtract Error from $\bar{x}$, to obtain the lower bound of left side of the interval.

26) and 27): These are both T-intervals because we are given small samples; therefore, the population standard deviation is not known. We realize that the sample mean $(\bar{x})$ and the sample standard deviations are not given. That is why the first step consists of finding both, $\bar{x}$ and s.

Most calculators, including non-graphing calculators, are able to handle these calculations. Once you find the sample mean and the sample standard deviation, these two questions 26 and 27, become the same type of questions we have seen before from 23 to 25. Follow the same procedure.

If you are using a graphing calculator, you may enter the given data on L1; then proceed to T-Interval, and choose, for TIs, Data, hit enter. Keep L1, choose the confidence level and calculate. The screen looks as follows:

```
TInterval
 Inpt:DATA Stats
 List:L₁
 Freq:1
 C-Level:.95
 Calculate
```

In Casio graphing calculator: enter your data in L1, then choose t-interval and press F1 for List:

```
1-Sample tInterval
Data     :Variable
C-Level :0.95
x̄        :0
sx        :1
n         :0
Save Res:None
List Var
```

and update the confidence level:

```
1-Sample tInterval
Data     :List
C-Level :0.95
List     :List1
Freq     :1
Save Res:None
Execute
List Var
```

28) and 29): these two questions are about finding the sample size for means. The formula is:

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

Substitute in the values and round up the result. Recall that $Z_{\alpha/2}$ for 95% = 1.96; for 99% = 2.575

29) and 30): These two questions σ is given; therefore, we deal with Z-Intervals. Follow the instructions for Z-Intervals in graphing calculators or use the following formula:

$E = \dfrac{\sigma}{\sqrt{n}}$ to calculate the Error; and then, add the error to the sample mean $\bar{x}$ which yields the upper bound or right side of the interval; and subtract Error from $\bar{x}$, to obtain the lower bound of left side of the interval.