1) Use the formula for mean of a discrete probability distribution: \( \mu = \sum x \cdot px \) or use a calculator. On the calculator, \( x \), values of the random variable correspond to L1, and probabilities, \( p(x) \) correspond to L2.
Using the formula: \( \mu = 0(0.23) + 1(0.20) + 2(0.18) + 3(0.24) + 4(0.13) = 1.68 \)

2) Using calculators, same procedure as previous question.
Using the formula: \( \sigma = \sqrt{\sum (x - \mu)^2 \cdot px} \) or use the shortcut formula: \( \sigma = \sqrt{\sum x^2 \cdot px} - \mu^2 \)
Find the mean first: \( \mu = 1.86 \)
Now, using the shortcut formula: \( \sigma = \sqrt{0(0.19) + 1(0.26) + 4(0.18) + 9(0.24) + 16(0.13) - 1.86^2} \approx 1.33 \)

3) Create a table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( px )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.15</td>
</tr>
<tr>
<td>5</td>
<td>0.30</td>
</tr>
<tr>
<td>6</td>
<td>0.25</td>
</tr>
<tr>
<td>7</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Find the mean, \( \mu = 5.7 \) and the standard deviation, \( \sigma = 1.05 \). Minimum usual number is given by \( \mu - 2 \cdot \sigma = 5.7 - 2(1.05) = 3.6 \); the max usual number of games is: \( \mu + 2 \cdot \sigma = 5.7 + 2(1.05) = 7.8 \); therefore, 7 is not an unusually high number of games (7 lies in between min and max).

4) Graphing calculators: binomial pdf, where number of trials, \( n = 12 \), \( p = 0.25 \) and number of successes, \( x = 5 \).
Formula: \( P(5) = 12C5 \cdot (0.25)^5(0.75)^7 = 0.1032 \)

5) \( P \) (at least nine)=\( P(9) + P(10) \)
Graphing calculators: binomial pdf with \( n = 10 \), \( p = 1/2 \) \( x = 9 \) in order to find probability of nine successes; then repeat the calculation for 10 successes, add up the two results: 0.009765 + 0.0009765 = 0.0107 \( \approx 0.011 \)
Formula: \( P(9) = 10C9 \cdot (0.5)^9(0.5) = 0.009765 \quad P(10) = 10C10 \cdot (0.5)^{10}(0.5)^0 = 0.009765 \)
And, again, add up the two results: 0.009765 + 0.0009765 = 0.0107 \( \approx 0.011 \)

6) \( P \) (3 or fewer)=\( P(0)+P(1)+P(2)+P(3) \) where \( n = 12 \), \( p = 0.38 \), \( q = 1 - 0.38 = 0.62 \) Using the formula:
\( P(0)=12C0(0.38)^0(0.62)^{12} = 0.0032 \)
\( P(1)=12C1(0.38)^1(0.62)^{11} = 0.0237 \)
\( P(2)=12C2(0.38)^2(0.62)^{10} = 0.0799 \)
\( P(3)=12C3(0.38)^3(0.62)^9 = 0.1634 \)
Adding up the results, yields: \( P \) (3 or fewer) =0.2704. By graphing calculator: use Bin cdf with \( n = 12 \), \( p = 0.38 \), and \( x = 3 \).
7) Calculator: Binom pdf with $n = 8$ (number of trials) $p = 0.46$ (probability of success in a single trial) and $x = 3$, number of success.

Formula: Considering that $q = 1 - 0.46 = 0.54$; then, $P(3) = \binom{8}{3} \cdot (0.46)^3(0.54)^5 = 0.2503$

8) Calculator, binom pdf: $n = 7$, $p = 1/5 = 0.2$ $x = 3$. Formula: $q = 1 - 0.2 = 0.8$; therefore, $P(3) = \binom{7}{3}(0.2)^3(0.8)^4 = 0.1147$

9) Given $n = 166$, $p = 0.15$ calculate $q = 1 - 0.15 = 0.85$. Find the mean ($\mu$) and the standard deviation of the binomial random variable:

$\mu = n \cdot p = 166 \cdot 0.15 = 24.9$ and $\sigma = \sqrt{166(0.15)(0.85)} = 4.6$.

Minimum is given by: $\mu - 2\sigma = 24.9 - 2(4.6) = 15.7$

Maximum is given by: $\mu + 2\sigma = 24.9 + 2(4.6) = 34.1$

10) Uniform distributions. Probability is calculated finding the area of the corresponding rectangle; that is: Area = probability = 10.

Between 2.3 and 5 the length $= 5 - 2.3 = 2.7$ Then $A = 2.7 \cdot 0.125 = 0.3375$

11) Uniform distribution. Draw a rectangle with length equal to six (the variable in consideration changes from 6 to 12), the height is the reciprocal of the length, or 1/6.

Less than 11 represents a change form 6 to 11 or a length of 5 units. Therefore,

Probability = Area $= 5 \cdot 1/6 = \frac{5}{6}$

12) Question gives us as a premise that $z$ is a standard normal variable; therefore, $\mu = 0$ and $\sigma = 1$.

Use the $z$ score table or a graphing calculator. On the table, locate $z = 1.13$, the area on the left of $z = 1.13$ is 0.8708.

On a calculator, go to distr (distributions, select normal cdf and enter as lower bound $-E99$ or simply $-999$, as upper bound 1.13, $\mu = 0$ and $\sigma = 1$. It yields 0.87076 or 0.8708 correct to four decimal places.

13) Again, standard normal distribution. Locate both $z$ scores on the table, and find the difference between the areas. The answer is 0.2237.

Using a graphing calculator, go to distrubitions, normal cdf, lower is $-1.10$, upper is $-0.36$.

14) The variable (IQ scores) it is normal distributed, but this is not a standard normal variable; so, this time if we use table, we need to find the $z$ score. $z = \frac{x - \mu}{\sigma}$; let’s calculate the $z$ scores for both values (90 and 120), once we have the $z$ scores, the procedure is the same we followed on question 13. Find the areas in the table, and the difference between the areas.

For $x = 90$, $z = \frac{90 - 100}{15} = -0.67$ and for $x = 120$, $z = \frac{120 - 100}{15} = 1.33$

Answer: Area $= 0.9082 - 0.2514 = 0.6568$.

15) $P_{10}$ stands for tenth percentile or area $= 0.10$

Using tables: find the area 0.1000 inside the $z$ score table. The closest value is 0.1003, that corresponds to a $z$ score of $-1.28$.

Solving for $x$ from $z = \frac{x - \mu}{\sigma}$, $x = z\sigma + \mu = -1.28(15) + 100 = 80.8$.

Using a graphing calculator, InvNorm, Area $= 0.10$, $\mu = 100$, $\sigma = 15$, yields 80.8.

16) Same procedure as question number 14.

17) In this question and the next, we don’t compare a value with respect to the population mean; but a sample mean (there is a sample of size equal to 90 students). The Central Limit Theorem applies. The formula for the $z$ score becomes:

$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{53.527 - 53}{\frac{5}{\sqrt{90}}} = 1.00$ the corresponding area on the $z$ score table is 0.8413; since the questions asks for at least 53.527 we need to find the prob of that value or greater; therefore, the answer is $1 - 0.8413 = 0.1587$. 

2
Using a graphing calculator, enter lower bound as 53.527, upper bound as E99 or 9999, \( \mu = 53 \) and \( \sigma = \frac{5}{\sqrt{90}} \). The result is 0.1587.

18) Central Limit Theorem applies: \( z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{215 - 200}{\frac{50}{\sqrt{40}}} = 1.90 \) On the table the corresponding area is 0.9713; since we need the prob of above that value, the answer is \( 1 - 0.9713 = 0.0287 \).

On a graphing calculator, go to Normal cdf, enter lower bound as 215, upper bound as E99 or 9999, \( \mu = 200, \sigma = \frac{5}{\sqrt{40}} \). It yields 0.02888. Since the calculator does not drop decimals, this is the exact answer; the answer choice is D) 0.0287, obtained using the z score table.

Confidence intervals:

19) Using a graphing calculator you may find the interval, say for TI 83 or 84, STAT, TEST, 1-PropZint; update \( x, n \) and confidence level. It yields (0.2852, 0.3148), and then calculate the margin of Error using: \( E = \frac{0.3148 - 0.2852}{2} = 0.0148 \)

Using the formula for margin for Error (one proportion): \( E = Z_{\alpha/2} \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}} = 2.575 \sqrt{\frac{0.30 \cdot 0.70}{6400}} = 0.0148 \)

Recall that \( \hat{p} = \frac{x}{n} = \frac{1920}{6400} = 0.30 \) and \( \hat{q} = 1 - \hat{p} = 1 - 0.30 = 0.70 \); and the \( Z_{\alpha/2} \) for 99% is 2.575

20) Using a graphing calculator, follow the instructions on how to create Confidence Intervals for one proportion.

On imathles.com: Instructions on how to construct CIs using TI83 and 84; or, Instructions on how to construct CIs using Casio9750GII and 9860GII.

Using formulas: First, find \( \hat{p} = \frac{x}{n} \); then, \( \hat{q} = 1 - \hat{p} \); use the specific \( Z_{\alpha/2} \) for 95% = 1.96 and use the formula for Error:

\[ E = Z_{\alpha/2} \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}} \]

once we know the Error, add the Error to \( \hat{p} \) that yields the right side or upper bound of the interval; and subtract the Error from \( \hat{p} \), in order to obtain the lower bound or left side of the interval. This procedure is described by the formula \( \hat{p} \pm E \).

For 20) \( \hat{p} = \frac{x}{n} = \frac{124}{171} = 0.7251 \) and \( \hat{q} = 1 - \hat{p} = 1 - 0.7251 = 0.2749 \) \( E = Z_{\alpha/2} \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}} = 1.96 \sqrt{\frac{0.7251 \cdot 0.2749}{171}} = 0.0669 \)

\( \hat{p} \pm E \quad 0.7251 \pm 0.0669 \quad (0.658, 0.792) \)

21) In order to find minimum sample size for proportions. Use the formula: \( n = \frac{[Z_{\alpha/2}]^2 \cdot \hat{p} \cdot \hat{q}}{\frac{E^2}{2}} \)

Whenever \( \hat{p} \) and \( \hat{q} \) are unknown, we enter 0.50 for each of them; so, the product of the two values is 0.25; again, whenever they are unknown; otherwise, enter the given values; as a rule only \( \hat{p} \) is given, find \( \hat{q} = 1 - \hat{p} \).

The \( Z_{\alpha/2} \) for 99% = 2.575; for 95% = 1.96

Remember that the result of the calculation is always round up to the next integer.

22) \( \hat{p} = 0.87 \); therefore, \( \hat{q} = 0.13 \), since \( Z_{\alpha/2} = 1.96 \), the sample size is given by:

\[ n = \frac{[Z_{\alpha/2}]^2 \cdot \hat{p} \cdot \hat{q}}{\frac{E^2}{2}} = \frac{1.96^2(0.87)(0.13)}{0.09^2} \approx 54. \]

23) Same type of question as question number 20. In this instance, \( n = 865, \quad x = 408, \quad 95\% \) Confidence interval.

24) Confidence intervals for sample mean. Since in this cases \( \sigma \) is not given, but \( s \) instead (the sample standard deviation), we construct a T-Interval. Follow the procedure in a graphing calculator or, use the formula:

First, find the Error: \( E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \)

In order to find \( t_{\alpha/2} \) use the t table of critical values, look up degrees of freedom equal to one less than \( n \). For \( n = 10 \) \( df = 10 - 1 = 9 \) and the the corresponding confidence level. Once the Error is calculated, add and subtract the error from the sample mean \( \bar{x} \)to obtain the lower bound and the upper bound of the interval.

Calculations using formulas:

\[ E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = 2.262 \cdot \frac{3.7}{\sqrt{10}} = 2.6466 \]

\( \bar{x} \pm E \quad 12.7 \pm 2.6466 \quad (10.053, 15.346) \)
25) Same procedure as question 24.

\[ E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = 2.01 \cdot \frac{10.5}{\sqrt{12}} = 6.6714 \]
\[ \bar{x} \pm E \]
\[ 198 \pm 6.6714 \]
\[ (191.328, 204.671) \]

26) T-interval because we are given small samples; therefore, the population standard deviation is not known. We realize that the sample mean (\( \bar{x} \)) and the sample standard deviations are not given. That is why the first step consists of finding both, \( \bar{x} \) and s.

Most calculators, including non-graphing calculators, are able to handle these calculations. Once you find the sample mean and the sample standard deviation, this question becomes the same type of questions we have seen before (same as number 25). Follow the same procedure.

If you are using a graphing calculator, you may enter the given data on L1; then proceed to T-Interval, and choose, for TIs, Data, hit enter. Keep L1, choose the confidence level and calculate. The screen looks as follows:

```
TInterval
Input: Data Stats
List:L1
Freq:1
C-Level: .95
Calculate
```

In Casio graphing calculator: enter your data in L1, then choose t-interval and press F1 for List:

```
1-Sample tInterval
Data: 3 Variable
C-Level: .95
\( \bar{x} \): 10
s: 41
n: 10
Save: Rest: None
List: Var
```

and update the confidence level:

```
1-Sample tInterval
Data: 3 Variable
C-Level: .95
List: L1
Freq: 1
Save: Rest: None
Execute
List: Var
```

27) and 28): These two questions either \( \sigma \) is given [#27], or the sample size is large enough [#28, \( n = 136 \)]; therefore, we deal with Z-Intervals. Follow the instructions for Z-Intervals in graphing calculators. Or, use the following formula:

\[ E = \frac{\sigma}{\sqrt{n}} \]

to calculate the Error; and then, add the error to the sample mean \( \bar{x} \) which yields the upper bound or right side of the interval; and subtract Error from \( \bar{x} \), to obtain the lower bound of left side of the interval.

For 27)
\[ E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 1.645 \cdot \frac{36}{\sqrt{108}} = 5.6984 \]
\[ \bar{x} \pm E \]
\[ 547 \pm 5.6984 \] The interval is: \( (541.30, 552.698) \)

29) and 30): These two questions are about finding the sample size for means. The formula is:

\[ n = \left( \frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2 \]

Substitute in the values and round up the result. Recall that \( Z_{\alpha/2} \) for 99% = 2.575; for 90% = 1.645

For 29)
\[ n = \left( \frac{2.575 \cdot 584}{136} \right)^2 = 122.26 = 123 \]

For 30)
\[ n = \left( \frac{1.645 \cdot 27}{2.8} \right)^2 = 251.6 = 252 \] The key word within refers to error.