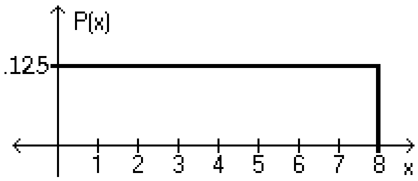
STA2023 . Chapter 6: The Standard Normal distribution.

Practice 6. Broward College.

Answers from imathesis.com.
Comments? email csotuyo@imathesis.com

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Using the following uniform density curve, answer the question.



This is a uniform density distribution.
Probabilities are calculated by finding Areas = length * height

1) What is the probability that the random variable has a value greater than 5? 1) D

A) 0.250 B) 0.325 C) 0.500 D) 0.375

Prob x>5 = length * height = 3 * 0.125 = 0.375

2) What is the probability that the random variable has a value greater than 3.3? 2) A

A) 0.5875 B) 0.4625 C) 0.5375 D) 0.7125

P(x>3.3)= 4.7 * 0.125 = 0.5875 Note that length = 8-3.3 = 4.7

3) What is the probability that the random variable has a value less than 3? 3) C

A) 0.250 B) 0.500 C) 0.375 D) 0.125

P(x<3)= 3 * 0.125 = 0.375

4) What is the probability that the random variable has a value greater than 1? 4) A

A) 0.875 B) 0.825 C) 1.000 D) 0.750

x>1 = 8-1 = 7 P(x>1)= 7 * 0.125 = 0.875

5) What is the probability that the random variable has a value greater than 1.4? 5) C

A) 0.7750 B) 0.9500 C) 0.8250 D) 0.7000

x>1.4 = 8 - 1.4 = 6.6 P(x>1.4)=6.6*0.125 = 0.825

6) What is the probability that the random variable has a value less than 5? 6) B

A) 0.375 B) 0.625 C) 0.500 D) 0.750

P(x<5)= 5 * 0.125 = 0.625

Assume that the weight loss for the first month of a diet program varies between 6 pounds and 12 pounds, and is spread evenly over the range of possibilities, so that there is a uniform distribution. Find the probability of the given range of pounds lost. Note: Think of a rectangle of length from 6 to 12; height is the reciprocal of length, that is 1/6.

7) More than 10 pounds 7) A

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{1}{7}$ D) $\frac{5}{6}$

P(x>10) = 2 * 1/6 = 1/3

8) Between 7 pounds and 10 pounds 8) D

A) $\frac{1}{3}$ B) $\frac{1}{4}$ C) $\frac{2}{3}$ D) $\frac{1}{2}$

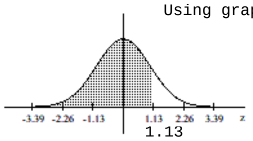
P(7 < x < 10)= 3 * 1/6 = 1/2

Find the area of the shaded region. The graph depicts the standard normal distribution with mean 0 and standard deviation 1.

9) Using table for Normal distribution, locate z = 1.13 Area given by table (always "left" or less than x)= 0.8708

9) C ______

Using graphing Calc, Normal cdf: lower is -9999 or -E99, upper = 1.13 mean = 0, sd=1.



Note: For questions 9, 10 and 11, on the calculator, we use normal CDF.

A) 0.8485 B) 0.1292 C) 0.8708 D) 0.8907

10)

10) D ______

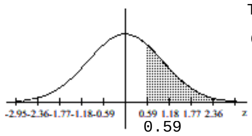


Table: locate z = 0.59; are given, left side,=0.7224; are on the right, greater than 0.59 = 1 - 0.7224 = 0.2776

Calculator: Lower = 0.59; upper = 9999 or E99, mean = 0; SD = 1.

A) 0.7224 B) 0.2224 C) 0.2190 D) 0.2776

11)

11) B ______

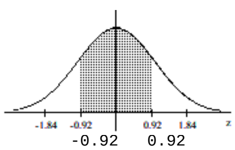


Table: find area A2 for z =0.92 which is 0.8212; then A1 for z= -0.92 equal to 0.1788; the Area in between the two is the difference of these two areas = 0.8212 - 0.1788 =0.6424

Calculator: Lower = -0.92; upper = 0.92, mean = 0, SD = 1.

A) 0.1788 B) 0.6424 C) 0.8212 D) 0.3576

Find the indicated z score. The graph depicts the standard normal distribution with mean 0 and standard deviation 1.

12) Shaded area is 0.9599.

12) D ______

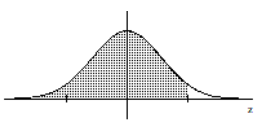
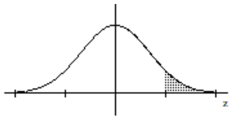


Table: locate "inside" the table this probability (or area) = 0.9599, by checking row and column we learn that the z score that correspond to this area is 1.75

Calculator: In this case we know the probability,same as area, same as "percentile". We need to use Inverse Normal. Enter the area = 0.9599, mean = 0, SD = 1. Done.

A) -1.38 B) 1.82 C) 1.03 D) 1.75

13) Shaded area is 0.0694. 13) A



This time the given area is to the right of the z score. Tables and most calculators only give us area to the left of the z score. Since the total area = 1, we find 1 - given area: 1 - 0.0694 = 0.9306
Using the table locate that number "inside" the table, the corresponding z score is 1.48
In Calculators, for area enter 1-0.0694, for mean = 0, SD = 1. Hit enter. Done.

A) 1.48 B) 1.45 C) 1.26 D) 1.39

If z is a standard normal variable, find the probability. Notice that for z scores mean = 0 & SD = 1. Always...

14) The probability that z lies between 0 and 3.01 14) D

A) 0.5013 B) 0.1217 C) 0.9987 D) 0.4987

Table: Find area for z = 3.01 (.9987) and for 0 (0.5000); find the difference of the two areas = 0.4987. Calculators: normal CDF, lower =0; upper = 3.01 mean = 0, SD = 1.

15) The probability that z lies between -2.41 and 0 15) B

A) 0.0948 B) 0.4920 C) 0.4910 D) 0.5080

Table: Area for z= -2.41 (0.0080) for z = 0 (0.5000); difference = 0.4920 [Always larger area minus smaller area]. Calc: lower = -2.41, upper=0; mean=0, SD=1.

16) The probability that z is less than 1.13 16) B

A) 0.8485 B) 0.8708 C) 0.1292 D) 0.8907

Table: Locate z = 1.13; "less than" is "left" areas given by tables. The corresponding area is 0.8708; in Calc; lower = -9999 or -E99, upper =1.13, mean=0, SD=1.

Find the indicated value.

Note: The notation in which the number is a subscript of Z indicates "the Z that correspond to an alpha value of 0.005"

17) $z_{0.005}$ 17) C

A) 2.015 B) 2.835 C) 2.575 D) 2.535

Alpha values indicate the area of the right tail of the normal curve. Tables give us the left, so say 1-0.005=0.995 and search for the z score that correspond to 0.995, which is 2.575 (approx); using Calc, InvNorm of area = 1-0.005 = 0.995, mean=0, SD=1, retrieves 2.5758.

Provide an appropriate response.

18) Assume that adults have IQ scores that are normally distributed with a mean of 100 and a standard deviation of 15 (as on the Wechsler test). Find the probability that a randomly selected adult has an IQ between 90 and 120 (somewhere in the range of normal to bright normal). 18) A

Calculators: normalcdf, lower 90, upper 120, mean =100, SD=15. Hit enter.

A) 0.6568 B) 0.6014 C) 0.6227 D) 0.6977

Tables; Find z score for x = 90; then, the z score for x =120. Recall: z = (x-mean)/SD
Z for x=90 = (90-100)/15 = -0.67 For 120, z=(120-100)/15=1.33 Area for z=-0.67 = 0.2514; for z=1.33, A = 0.9082: the diff between the two = 0.6568

19) Assume that adults have IQ scores that are normally distributed with a mean of 100 and a standard deviation of 15 (as on the Wechsler test). Find P_{10}, which is the IQ score separating the bottom 10% from the top 90%. 19) D

For #19. P10 same as 10% = 0.1000; the corresponding z = -1.28; Therefore, x = -1.28 * 15+100 = 80.8
Calculator: InvNorm, Area = 0.10; mean=100; SD = 15

A) 81.9 B) 81.3 C) 80.1 D) 80.8

From 19 to 23 percentiles are given. Using the "bottom" area or left area, we find corresponding Z on table; then solve for x = Z* SD + mean; in calculators we reach Inv Norm enter the "bottom" or left area, mean & SD: hit enter.

20) Assume that adults have IQ scores that are normally distributed with a mean of 100 and a standard deviation of 15 (as on the Wechsler test). Find the IQ score separating the top 16% from the others. 20) C

A) 85.0 B) 108.1 C) 114.9 D) 99.1

Top 16%, therefore "bottom"is 100-16 =84%. As decimal, 0.8400; corresponding Z is 0.99; therefore, x = 0.99*15+100=114.85
In Calculators: InvNorm, area = 0.84, mean=100, SD=15. Done.

Solve the problem. Round to the nearest tenth unless indicated otherwise.

21) In one region, the September energy consumption levels for single-family homes are found to be normally distributed with a mean of 1050 kWh and a standard deviation of 218 kWh. Find P_{45}, which is the consumption level separating the bottom 45% from the top 55%. 21) D

A) 1078.3 B) 1148.1 C) 1087.8 D) 1021.7

Bottom 45%, = 0.45000; corresponding z = -0.13. x = -0.13 * 218 +01050 = 1021.66 Calculator: Inv Norm, area 0.45; mean = 1050, SD = 218.

22) Scores on a test are normally distributed with a mean of 68.9 and a standard deviation of 11.6. Find P_{81}, which separates the bottom 81% from the top 19%. 22) B

A) 72.3 B) 79.1 C) 0.88 D) 0.291

Bottom 81% = 0.8100, by Table corresponding z = 0.88 x = 0.88*11.6 + 68.9 = 79.108 Calculators: Inv Norm: area = 0.81, mean = 68.9, SD = 11.6

23) A bank's loan officer rates applicants for credit. The ratings are normally distributed with a mean of 200 and a standard deviation of 50. Find P_{60}, the score which separates the lower 60% from the top 40%. 23) A

A) 212.5 B) 211.3 C) 207.8 D) 187.5

Bottom 60%, = 0.6000 by Table corresponding z=0.25 x=0.25*50+200=212.50 Calculator: Inv Norm: area=0.60, mean = 200, SD=50.

Assume that X has a normal distribution, and find the indicated probability.

Beginning #24 we need to find z scores; since mean is not 0, SD not = 1. Z score formula: Z = (x - mean)/SD

24) The mean is $\mu = 60.0$ and the standard deviation is $\sigma = 4.0$. Find the probability that X is less than 53.0. 24) D

A) 0.0802 B) 0.9599 C) 0.5589 D) 0.0401

z=(53-60)/4 = -1.75 Check table for area corresponding z=-1.75, tables give us area to the left of value, which is less than the value = 0.0401
Calculator: Normal cdf, lower= -9999 or -E99, upper = 53, mean = 60, SD = 4.

25) The mean is $\mu = 15.2$ and the standard deviation is $\sigma = 0.9$. Find the probability that X is greater than 15.2. 25) C

A) 1.0000 B) 0.9998 C) 0.5000 D) 0.0003

Z= (15.2 -15.2)/0.9 = 0; Z = 0 is located right in the middle = 0.5000. Calculator: normal cdf: lower = 15.2, upper = 9999; mean = 15.2, SD = 0.90
Notice that 0 is right on the middle of the Normal curve; so, the area to the left is 0.5000; on the right, the other half = 0.5000.

26) The mean is $\mu= 15.2$ and the standard deviation is $\sigma = 0.9$. Find the probability that X is greater than 16.1. 26) C

Calculator: normal cdf, lower 16.1 upper, 9999; mean:15.2 SD: 0.9

A) 0.1357 B) 0.1550 C) 0.1587 D) 0.8413

Z= (16.1-15.2)/0.90 = 1.0 By table, area to left of Z = 1 is 0.8413; But, in this case we need to find prob of "greater than" or, to the right of x = 16.1; therefore, since table gives us area on the left, the right area = 1 - 0.8413 = 0.1587

Find the indicated probability.

For 27 & 28, once the area -probability is found, mult *100 = percentage.

27) The diameters of bolts produced by a certain machine are normally distributed with a mean of 0.30 inches and a standard deviation of 0.01 inches. What percentage of bolts will have a diameter greater than 0.32 inches? 27) A

Calculator: normal cdf: lower = 0.32, upper = 99999, mean = 0.30, SD = 0.01

A) 2.28% B) 97.72% C) 37.45% D) 47.72%

Find z = (0.32-0.30)/0.01 = 2. Area to the left of 2 = 0.9772. To the right (greater than) = 1 - 0.9772 = 0.0228, now 0.0228 * 100 = 2.28%

28) The incomes of trainees at a local mill are normally distributed with a mean of $1100 and a standard deviation of $150. What percentage of trainees earn less than $900 a month? 28) C

Calculator: normalcdf, lower= -9999 upper= 900, mean = 1100, SD = 150.

A) 90.82% B) 40.82% C) 9.18% D) 35.31%

Z = (900- 1100)/150 = -1.33 Area on the left (see table) = 0.0918. Since question asks for "less than" that is "left" so the answer is 0.0918*100 = 9.18%

29) The volumes of soda in quart soda bottles are normally distributed with a mean of 32.3 oz and a standard deviation of 1.2 oz. What is the probability that the volume of soda in a randomly selected bottle will be less than 32 oz? 29) C

A) 0.3821 B) 0.5987 C) 0.4013 D) 0.0987

Z = (32 -32.3)/1.2 = -0.25 By table, area to the left of -0.25 is 0.4013. That is the answer because the question asks for "less than", and, as said above, less is "left", which is area given by tables. On Calculator: normalcdf, lower -9999; upper: 32 mean: 32.3, SD: 1.2

Solve the problem.

Note: problems 30, 31 & 32 we don't compare a value to the given mean, but a sample mean of sample size = n. The standard deviation must be divided by the square root of the sample size n. In #30 n = 49. Sqrt of 49 = 7. Then, 14/7 = 2.

30) The amount of snowfall falling in a certain mountain range is normally distributed with a mean of 89 inches, and a standard deviation of 14 inches. What is the probability that the mean annual snowfall during 49 randomly picked years will exceed 91.8 inches? 30) D

Calculator: normalcdf, lower: 91.8, upper: 9999, SD=14/7

A) 0.5808 B) 0.4192 C) 0.0026 D) 0.0808

Z =(91.8-89)/2 = 1.4, by table area = 0.9192; "exceeds" means greater, or area on the right: so, we have 1 - 0.9192 = 0.0808.

31) The annual precipitation amounts in a certain mountain range are normally distributed with a mean of 85 inches, and a standard deviation of 14 inches. What is the probability that the mean annual precipitation during 49 randomly picked years will be less than 87.8 inches? 31) A

Calculators:
Using SD = 14/Sqrt49 = 2
NormCDF, lower: -9999; upper: 87.8 mean = 85, SD = 2.

A) 0.9192 B) 0.5808 C) 0.4192 D) 0.0808

Again, sample size is 49. Sqrt 49 = 7. Dive SD by 7: 14/7 = 2. Find Z = (87.8-85)/2 = 1.4 Question asks for less, which is value given in tables: 0.9192.

32) The weights of the fish in a certain lake are normally distributed with a mean of 18 lb and a standard deviation of 12. If 16 fish are randomly selected, what is the probability that the mean weight will be between 15.6 and 21.6 lb? 32) B

A) 0.0968 B) 0.6730 C) 0.3270 D) 0.4032

In #32 sample size is 16. Sqrt 16 =4. Divide SD/4 = 12/4 = 3.
Finding Z scores for both x values: 15.6 and 21.6: Z = (15.6-18)/3 = -0.8 Z = (21.6 - 18)/3 = 1.2 The corresponding areas are, respectively 0.2119 and 0.8849, find the positive diff = 0.6730.

Calculators: Normcdf, lower 15.6, upper 21.6, mean 18 for SD, use 12/Sqrt 16 = 12/4 =3.

NOTE: Sqrt is short for "square root".