Notes on Probability

Basic Concepts:

Processes such as flipping a coin, rolling a die, or drawing a card from a deck are called probability experiments.

A probability experiment is a chance process that leads to well-defined results called outcomes.
An outcome is the result of a single trial of a probability experiment.
An event consists of a set of outcomes of a probability experiment.
Equally likely events are events that have the same probability of occurring.

There are three basic interpretations of probability:

1. Classical probability
2. Empirical or relative frequency probability
3. Subjective probability

Formula for Classical Probability:

The probability of any event E is $P(E) = \frac{nE}{nS}$ where nE is number of outcomes of event E and nS is the total number of outcomes in the sample space.

Probability Rule 1:
The probability of any event E is a number (either a fraction or decimal) between and including 0 and 1.

Probability Rule 2:
If an event E cannot occur its probability is 0.

Probability Rule 3:
If an event E is certain, then the probability of E is 1.

Probability Rule 4:
The sum of the probabilities of all the outcomes in the sample space is 1.

Probability Rule 5:

$p(E) + p\bar{E} = 1$

Where $p\bar{E}$ denotes the complement of an event E --- the set of outcomes in the sample space that are not included in the outcomes of event E.

Formula for Empirical Probability:

Given a frequency distribution, the probability of an event being is given by

$$pE = \frac{f}{n}$$

This probability is called empirical probability and is based on observation.

Subjective Probability:

The third type of probability is called subjective probability. Subjective probability uses a probability value based on an educated guess or estimate, employing opinions and inexact information.

The Addition Rules for Probability:

Two events are mutually exclusive events if they cannot occur at the same time.

When two events A and B are mutually exclusive, the probability that A or B will occur is

$$P(A \ \ or \ \ B) = P(A) + P(B)$$

If A and B are not mutually exclusive, then:

$$P(A \ or \ B) = P(A) + P(B) - P(A \ and \ B)$$

The Multiplication Rules and Conditional Probability:

The multiplication rules can be used to find the probability of two or more events that occur in sequence.

Example: roll a die and toss a coin. What is the probability of obtaining a 5 and head, in that order? Create a probability *tree.*

When two events are independent, the probability of both occurring is

$$P(A \ and \ B) = P(A) \ * \ P(B)$$

In the previous example the events are independent (the first outcome does not change the probability of the second outcome).

When the outcome or occurrence of the first event affects the outcome or occurrence of the second event in such a way that the probability is changed, the events are said to be dependent events.

Example of **dependent** events:

a. Drawing a card from a deck, not replacing it, and then drawing a second card.

b. Selecting a ball from an urn, not replacing it, and then selecting a second ball.

c. Parking in a no-parking zone and getting a parking ticket.

The conditional probability of an event B in relationship to an event A is the probability that event B occurs after event A has already occurred. The notation for conditional probability is P(B|A).

Multiplication Rule for dependent events:

When two events are dependent, the probability of both occurring is

$$P(A\ and\ B) = P(A)\ *\ P(B|A)$$

Dividing by P(A) we obtain the Formula for Conditional Probability

$$P(B|A) = \frac{p(A\ and\ B)}{p(A)}$$

Probabilities for *At Least*:

P(at least 1 of event E) = 1 – P (none of event E)

Example: A coin is tossed 3 times. Find the probability of getting at least 1 tail.

Counting Rules:

Fundamental Counting Rule

In a sequence of n events in which the first one has k_1 possibilities ***and*** the second event has k_2 ***and*** the third has k_3, ***and*** so forth, the total number of possibilities of the sequence will be

$$k_1 * k_2 * k_3 \dots k_n$$

Note: In this case ***and*** means to multiply.

In how many way n distinct objects may be arranged?

$n!$ n factorial ways. Explain

The arrangement of n objects in a specific order using r objects at a time is called a permutation of n objects taking r objects at a time. It is written as nPr, and the formula is:

$$nPr = \frac{n!}{(n-r)!}$$

A selection of distinct objects without regard to order is called a combination.

The number of combinations of r objects selected from n objects is denoted by nCr and is given by the formula:

$$nCr = \frac{n!}{(n-r)!\,r!}$$