

## Comparing two groups

Studies often compare two groups. For example, researchers are interested in the effect aspirin has in preventing heart attacks. Over the last few years, newspapers and magazines have reported about various aspirin studies involving two groups. Typically, one group is given aspirin and the other group is given a placebo. Then, the heart attack rate is studied over several years. There are other situations that deal with the comparison of two groups. For example, studies compare various diet and exercise programs. Politicians compare the proportion of individuals from different income brackets who might vote for them. Students are interested in whether SAT or GRE preparatory courses really help raise their scores. In this course, we have learned to conduct hypothesis tests on single means and single proportions. In this chapter we will compare two averages or two proportions to each other. The general procedure is still the same, just expanded.

**To compare two averages or two proportions, you work with two groups.** The groups are classified either as independent or matched pairs. Independent groups mean that the two samples taken are independent, that is, sample values selected from one population are not related in any way to sample values selected from the other population. Matched pairs consist of two samples that are dependent. The parameter tested using matched pairs is the population mean. The parameters tested using independent groups are either population means or population proportions. NOTE: This chapter relies on either a calculator or a computer to calculate the degrees of freedom, the test statistics, and p-values. This chapter deals with the following hypothesis tests: Independent groups (samples are independent) • Test of two population means. • Test of two population proportions. • Matched or paired samples (samples are dependent) which becomes a test of one population mean.

### Comparing Two Independent Population Means with Unknown Population Standard Deviations

The two independent samples are simple random samples from two distinct populations. Both populations are normally distributed with the population means and standard deviations unknown unless the sample sizes are greater than 30. In that case, the populations need not be normally distributed. The comparison of two population means is very common. A difference between the two samples depends on both the means and the standard deviations. Very different means can occur by chance if there is great variation among the individual samples. In order to account for the variation, we take the difference of the sample means,  $X_1 - X_2$ , and divide by the standard error in order to standardize the difference. The result is a t-score test statistic.

### Comparing Two Independent Population Means with Known Population Standard Deviations

Even though this situation is not likely (knowing the population standard deviations is not likely). The distribution is Normal and is for the difference of sample means,  $X_1 - X_2$ .

### Comparing Two Independent Population Proportions

The two independent samples are simple random samples that are independent. The number of successes is at least five and the number of failures is at least five for each of the samples. Comparing two proportions, like comparing two means, is common. If two estimated proportions are different, it may be due to a difference in the populations or it may be due to chance. A hypothesis test can help determine if a difference in the estimated proportions ( $P'A - P'B$ ) reflects a difference in the populations. The difference of two proportions follows an approximate normal distribution. Generally, the null hypothesis states that the two proportions are the same. That is,  $H_0 : p_A = p_B$ . To conduct the test, we use a pooled proportion,  $p_c$ .

### Matched or Paired Samples

1. Simple random sampling is used.
2. Sample sizes are often small
3. Two measurements (samples) are drawn from the same pair of individuals or objects.
4. Differences are calculated from the matched or paired samples.
5. The differences form the sample that is used for the hypothesis test.
6. The matched pairs have differences that either come from a population that is normal or the number of differences is greater than 30 or both. In a hypothesis test for matched or paired samples, subjects are matched in pairs and differences are calculated. The differences are the data. The population mean for the differences,  $\mu_d$ , is then tested using a Student-t test for a single population mean with  $n - 1$  degrees of freedom where  $n$  is the number of differences.

Source: <http://www.webassign.net/idcollabstat2/Chapter10.pdf>