Chapter 8. Notes

Hypothesis Testing

- Hypothesis
  - In statistics, a **hypothesis** is a claim or statement about a property of a population.
- Hypothesis Test
  - A **hypothesis test** (or **test of significance**) is a procedure for testing a claim about a property of a population.

Using Technology It is easy to obtain hypothesis-testing results using technology. The accompanying screen displays show results from four different technologies, **so we can use computers or calculators to do all of the computational heavy lifting.**

Statdisk

Alternative Hypothesis:
p > p(hyp)

Sample proportion: 0.5401388
Test Statistic, z: 2.5500
Critical z: 1.6449
P-Value: 0.0054

90% Confidence interval:
0.5143312 < p < 0.5659463

Minitab

```
Test of p = 0.5 vs p > 0.5

Sample    X     N  Sample p  95% Lower Bound  Z-Value  P-Value
1       545  1009  0.540139         0.514331     2.55    0.005

Using the normal approximation.
```

TI-83/84 Plus

```
NORMAL FLOAT AUTO REAL RADIAN MP
1-PropZTest
prop>.5
z=2.549995628
p=.005386238
p̂=.5401387512
n=1009
```

StatCrunch

Hypothesis test results:
p : Proportion of successes
H_0 : p = 0.5
H_A : p > 0.5

Proportion	Count	Total	Sample Prop.	Std. Err.	Z-Stat	P-value
p	545	1009	0.54013875	0.015740714	2.5499956	0.0054

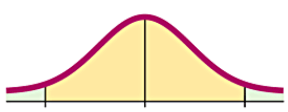
- Null Hypothesis
  - The **null hypothesis** (denoted by H_0) is a statement that the value of a population parameter (such as proportion, mean, or standard deviation) is **equal to** some claimed value.
- Alternative Hypothesis
  - The **alternative hypothesis** (denoted by H_1 or H_a or H_A) is a statement that the parameter has a value that somehow differs from the null hypothesis. For the methods of this chapter, the symbolic form of the alternative hypothesis must use one of these symbols: <, >, ≠.

The **significance level *α*** is the same *α* that defines a “critical value”. Common choices for *α* are 0.05, 0.01, and 0.10; 0.05 is most common.

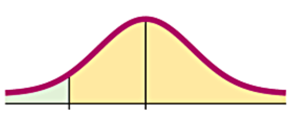
Parameter	Sampling Distribution	Requirements	Test Statistic
Proportion *p*	Normal (*z*)	$np \geq 5$ and $nq \geq 5$	$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$
Mean ***μ***	*t*	***σ*** not known and normally distributed population or ***σ*** not known and $n > 30$	$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$
Mean ***μ***	Normal (*z*)	***σ*** known and normally distributed population or ***σ*** known and $n > 30$	$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

Two-Tailed, Left-Tailed, Right-Tailed

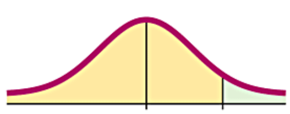
- **Two-tailed test:** The critical region is in the two extreme regions (tails) under the curve.
- **Left-tailed test:** The critical region is in the extreme left region (tail) under the curve.
- **Right-tailed test:** The critical region is in the extreme right region (tail) under the curve.



Sign used in H_1: $\neq$
Two-tailed test



Sign used in H_1: $<$
Left-tailed test



Sign used in H_1: $>$
Right-tailed test

- *P*-Value Method
  - In a hypothesis test, the ***P*-value** is the probability of getting a value of the test statistic that is **at least as extreme** as the test statistic obtained from the sample data, assuming that the null hypothesis is true.

Type I and Type II Errors

- **Type I error:** The mistake of rejecting the null hypothesis when it is actually true. The symbol α (alpha) is used to represent the probability of a type I error.

$\boldsymbol{\alpha}$ = ***P***(**type I error**) = ***P***(**rejecting H_0 when H_0 is true**)

- **Type II error:** The mistake of failing to reject the null hypothesis when it is actually false. The symbol β(beta) is used to represent the probability of a type II error.

$\boldsymbol{\beta}$ = ***P***(**type II error**) = ***P***(**failing to reject H_0 when H_0 is false**)

Preliminary Conclusion	**True State of Nature** **Null hypothesis is true**	**True State of Nature** **Null hypothesis is false**
Reject H_0	**Type I error:** Reject a true H_0.	Correct decision
Fail to reject H_0	Correct decision	**Type II error:** Fail to reject a false H_0.