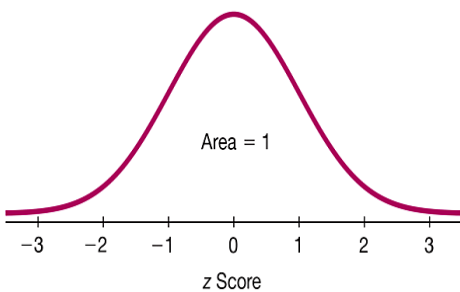


## Chapter 6 Notes

In this section we present the **standard normal distribution**, which is a specific normal distribution having the following three properties:

1. Bell-shaped: The graph of the standard normal distribution is bell-shaped.
2.  $\mu = 0$ : The standard normal distribution has a mean equal to 0.
3.  $\sigma = 1$ : The standard normal distribution has a standard deviation equal to 1.

Summary: The **standard normal distribution** is a normal distribution with the parameters of  $\mu = 0$  and  $\sigma = 1$ . The total area under its density curve is equal to 1.



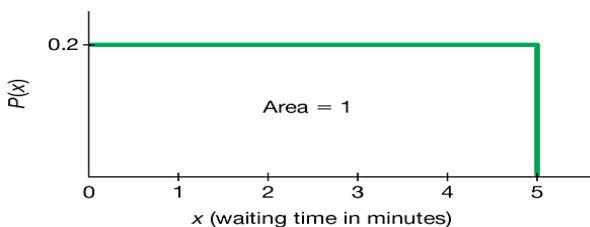
## Uniform Distribution

Properties of uniform distribution:

1. The area under the graph of a continuous probability distribution is equal to 1.
2. There is a correspondence between area and probability, so probabilities can be found by identifying the corresponding areas in the graph using this formula for the area of a rectangle:  
Area = height  $\times$  width

A continuous random variable has a **uniform distribution** if its values are spread **evenly** over the range of possibilities. The graph of a uniform distribution results in a rectangular shape.

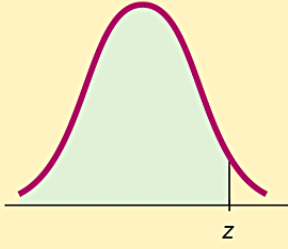
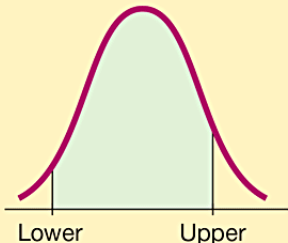
- Example: All of the different possible waiting times are **equally likely**.



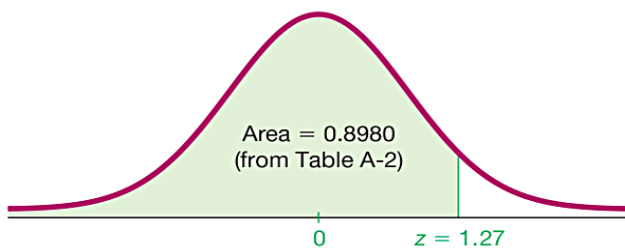
## Normal distribution: Finding Probabilities When Given z Scores

- We can find areas (probabilities) for different regions under a normal model using technology or Tables
- Technology is strongly recommended.

Because calculators and software generally give more accurate results than Tables, we **strongly** recommend using technology.

<p><b>Cumulative Area from the Left</b> The following provide the <i>cumulative area from the left</i> up to a vertical line above a specific value of <math>z</math>:</p> <ul style="list-style-type: none"><li>• <b>Table A-2</b></li><li>• <b>Statdisk</b></li><li>• <b>Minitab</b></li><li>• <b>Excel</b></li><li>• <b>StatCrunch</b></li></ul>	 <p>The diagram shows a normal distribution curve with the area to the left of a vertical line at <math>z</math> shaded in light green. The vertical line is labeled <math>z</math> on the horizontal axis.</p> <p><b>Cumulative Left Region</b></p>
<p><b>Area Between Two Boundaries</b> The following provide the area bounded on the left and bounded on the right by vertical lines above specific values.</p> <ul style="list-style-type: none"><li>• <b>TI-83/84 Plus calculator</b></li><li>• <b>StatCrunch</b></li></ul>	 <p>The diagram shows a normal distribution curve with the area between two vertical lines labeled 'Lower' and 'Upper' shaded in light green.</p> <p><b>Area Between Two Boundaries</b></p>

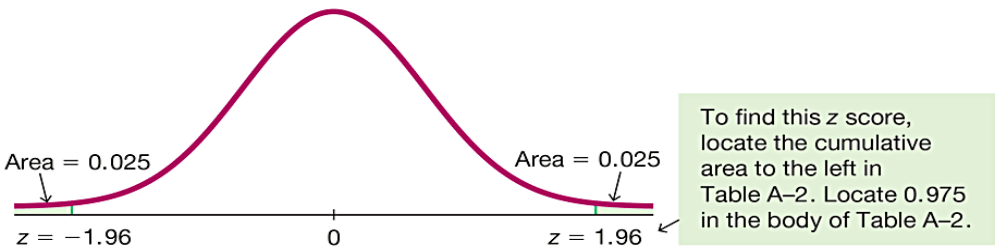
**Example:**  $p(z < 1.27) = 0.8980$



**Critical Value:**

For the standard normal distribution, a **critical value** is a z score on the borderline separating those z scores that are **significantly low** or **significantly high**.

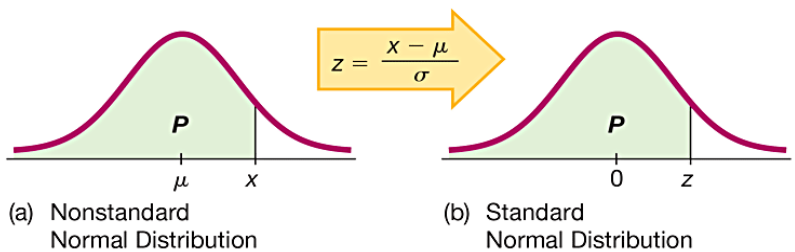
The expression  $z_\alpha$  denotes the z score with an area of  $\alpha$  to its right.



When working with normal distributions that are not standard. That is, the mean is not 0 or the standard deviation is not 1, or both.

The key is that we can use a simple conversion that allows us to “standardize” any normal distribution so that the same methods of the previous section can be used.

$$Z = \frac{X - \mu}{\sigma}$$



If you know z and must convert to the equivalent x value, use the conversion formula by entering the values for  $\mu$ ,  $\sigma$ , and the z score found in step 2, and then solve for x. We can solve for x as follows:

$$x = \mu + (z \cdot \sigma)$$

The **central limit theorem**:

The central limit theorem states that given a distribution with a mean  $\mu$  and variance  $\sigma^2$ , the sampling distribution of the mean approaches a normal distribution with a mean ( $\mu$ ) and a variance  $\sigma^2/n$  as  $n$ , the sample size, increases.

Consider  $n > 30$  a sufficiently large sample. Therefore, the **requirements**: is: **The population has a normal distribution or  $n > 30$** :

Then,

Mean of all values of  $\bar{x}$ :  $\mu_{\bar{x}} = \mu$

Standard deviation of all values of  $\bar{x}$ :  $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

z score conversion of  $\bar{x}$ :  $z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

**Check Requirements:** When working with the mean from a sample, verify that the normal distribution can be used by confirming that the original population has a normal distribution or the sample size is  $n > 30$ .

**Intro to t-** distribution.

**Intro to Non-**parametric statistics.