Chapter 4 Notes

Basic probability:

An **event** is any collection of results or outcomes of a procedure.

A **simple event** is an outcome or an event that cannot be further broken down into simpler components.

The **sample space** for a procedure consists of all possible **simple** events. That is, the sample space consists of all outcomes that cannot be broken down any further.

Classical Approach to Probability (Requires Equally Likely Outcomes) If a procedure has *n* different simple events that are **equally likely**, and if event *A* can occur in *s* different ways, then

$$P(A) = \frac{\text{number of ways } A \text{ occurs}}{\text{number of different simple events}} = \frac{s}{n}$$

– The **complement** of event *A*, denoted by $\bar{A}$, consists of all outcomes in which event *A* does **not** occur.

The Rare Event Rule for Inferential Statistics

If, under a given assumption, the probability of a particular observed event is very small and the observed event occurs **significantly less than or significantly greater than** what we typically expect with that assumption, we conclude that the assumption is probably not correct.

Borel Law?

Using Probabilities to Determine When Results Are Significantly High or Significantly Low

- **Significantly high number of successes:** *x* successes among *n* trials is a **significantly high** number of successes if the probability of *x* or more successes is unlikely with a probability of 0.05 or less. That is, *x* is a significantly high number of successes if *P*(*x* or more) ≤ 0.05*.

*The value 0.05 is not absolutely rigid.

ODDS:

– The **actual odds against** event *A* occurring are the ratio $\frac{P(\bar{A})}{P(A)}$, usually expressed in the form of *a*:*b* (or "*a* to *b*"), where *a* and *b* are integers. (Reduce using the largest common factor; if *a* = 16 and *b* = 4, express the odds as 4:1 instead of 16:4.)

– The **actual odds in favor** of event *A* occurring are the ratio $\frac{P(A)}{P(\bar{A})}$ which is the reciprocal of the actual odds against that event. If the odds against an event are *a*:*b,* then the odds in favor are *b*:*a*.

Addition rule:

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

where $P(A \text{ and } B)$ denotes the probability that *A* and *B* both occur at the same time as an outcome in a trial of a procedure.

Completment:

We use $\overline{A}$ to indicate that event *A* does not occur. Common sense dictates this principle: We are certain (with probability 1) that either an event *A* occurs *or* it does not occur, so it follows that $P(A \text{ or } \overline{A}) = 1$. Because events *A* and $\overline{A}$ must be disjoint, we can use the addition rule to express this principle as follows:

$$P(A \text{ or } \overline{A}) = P(A) + P(\overline{A}) = 1$$

Multiplication rule:

Notation

$P(A$ **and** $B)$ = *P*(event A occurs in a first trial and event B occurs in a second trial)

$P(B \mid A)$ represents the probability of event *B* occurring after it is assumed that event *A* has already occurred.

$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

- Independent
  - Two events *A* and *B* are **independent** if the occurrence of one does not affect the **probability** of the occurrence of the other. (Several events are independent if the occurrence of any does not affect the probabilities of the occurrence of the others.) If *A* and *B* are not independent, they are said to be **dependent.**

In the world of statistics, sampling methods are critically important, and the following relationships hold:

  - Sampling **with replacement:** Selections are **independent** events.
  - Sampling **without replacement:** Selections are **dependent** events.

- **5% Guideline for Cumbersome Calculations**
- When sampling without replacement and the sample size is no more than 5% of the size of the population, treat the selections as being **independent** (even though they are actually dependent).

The Probability of "At Least One"

Subtract the result from 1. That is, evaluate this expression:

P(**at least one** occurrence of event *A*) = 1 − *P*(**no** occurrences of event *A*)

Conditional Probability

A **conditional probability** of an event is a probability obtained with the additional information that some other event has already occurred.

Notation

$P(B \mid A)$ denotes the conditional probability of event *B* occurring, given that event *A* has already occurred.

The probability $P(B \mid A)$ can be found by dividing the probability of events *A* and *B* both occurring by the probability of event *A:*

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

Counting rules:

Factorial Rule

The number of different **arrangements** (order matters) of *n* different items when all *n* of them are selected is ***n*!**.

Permutations of items are arrangements in which different sequences of the same items are counted **separately.** (The letter arrangements of abc, acb, bac, bca, cab, and cba are all counted **separately** as six different permutations.)

Combinations of items are arrangements in which different sequences of the same items are counted as being the **same**. (The letter arrangements of abc, acb, bac, bca, cab, and cba are all considered to be the **same** combination.)

Permutations Rule

- When *n* different items are available and *r* of them are selected without replacement, the number of different permutations (order counts) is given by

$$_nP_r = \frac{n!}{(n-r)!}$$

Permutations Rule (When Some Items Are Identical to Others)

The number of different permutations (order counts) when *n* items are available and all *n* of them are selected **without replacement,** but some of the items are identical to others, is found as follows:

$$\frac{n!}{n_1!n_2! \ldots n_k!}$$

where n_1 are alike, n_2 are alike, . . . , and n_k are alike.

Combinations Rule:

When *n* different items are available, but only *r* of them are selected **without replacement,** the number of different combinations (order does not matter) is found as follows:

$$_nC_r = \frac{n!}{(n-r)!r!}$$