Section 1.4

Radical Equations; Equations Quadratic in Form; Factorable Equations
Example 1: Solving a Radical Equation
(1 of 2)

Find the real solutions of the equation:

$$\sqrt[3]{1 - 2x} - 1 = 0$$

The equation contains a radical whose index is 3. Isolate it on the left side.

$$\sqrt[3]{1 - 2x} = 1$$
Example 1: Solving a Radical Equation

(2 of 2)

\[\sqrt[3]{1 - 2x} = 1\]

Now raise both sides to the third power (the index of the radical is 3) and solve.

\[(\sqrt[3]{1 - 2x})^3 = 1^3\]

\[1 - 2x = 1\]

\[-2x = 0\]

\[x = 0\]

Raise both sides to the power 3.

Simplify.

Subtract 1 from both sides.

Divide both sides by −2.

Check: \[\sqrt[3]{1 - 2 \cdot 0} - 1 = \sqrt[3]{1} - 1 = 1 - 1 = 0\]

The solutions set is \{0\}. 

Example 2: Solving a Radical Equation
(1 of 3)

Find the real solutions of the equation:

\[ \sqrt{12 - 2x} = x - 2 \]

Square both sides since the index of a square root is 2.

\[ (\sqrt{12 - 2x})^2 = (x - 2)^2 \]

\[ 12 - 2x = x^2 - 4x + 4 \]

Square both sides.

\[ 0 = x^2 - 2x - 8 \]

Multiply out.

Put in standard form.
Example 2: Solving a Radical Equation

(2 of 3)

\[ x^2 - 2x - 8 = 0 \]
\[ (x + 2)(x - 4) = 0 \]
\[ x = -2 \quad \text{or} \quad x = 4 \]

Factor.

Use the Zero-Product Property and solve.

Check:

\[ x = -2 \quad \sqrt{12 - 2x} = \sqrt{12 - 2(-2)} = \sqrt{16} = 4 \]
and \[ x - 2 = -2 - 2 = -4 \]
\[ x = 4 \quad \sqrt{12 - 2x} = \sqrt{12 - 2(4)} = \sqrt{4} = 2 \]
and \[ x - 2 = 4 - 2 = 2 \]
Example 2: Solving a Radical Equation
(3 of 3)

The solution \(-2\) does not check, so it is extraneous; the only solution of the equation is 4. The solution set is \(\{4\}\).
Example 3: Solving a Radical Equation
(1 of 4)

Find the real solutions of the equation:

\[ \sqrt{2x + 3} - \sqrt{x + 1} = 1 \]

First, isolate the more complicated radical expression (in this case, \( \sqrt{2x + 3} \)) on the left side.

\[ \sqrt{2x + 3} = \sqrt{x + 1} + 1 \]

Now square both sides (the index of the isolated radical is 2).

\[ (\sqrt{2x + 3})^2 = (\sqrt{x + 1} + 1)^2 \quad \text{Square both sides.} \]
Example 3: Solving a Radical Equation

(2 of 4)

\[(\sqrt{2x+3})^2 = (\sqrt{x+1}+1)^2\]

\[2x + 3 = (\sqrt{x+1})^2 + 2\sqrt{x+1} + 1\]

Multiply out.

\[2x + 3 = x + 1 + 2\sqrt{x+1} + 1\]

Simplify.

\[2x + 3 = x + 2 + 2\sqrt{x+1}\]

Combine like terms.

Because the equation still contains a radical, isolate the remaining radical on the right side and again square both sides.

\[x + 1 = 2\sqrt{x+1}\]

Isolate the radical on the right side.
Example 3: Solving a Radical Equation

(3 of 4)

\[(x + 1)^2 = 4(x + 1)\]
\[x^2 + 2x + 1 = 4x + 4\]
\[x^2 - 2x - 3 = 0\]
\[(x - 3)(x + 1) = 0\]
\[x = 3 \text{ or } x = -1\]

Square both sides.

Multiply out.

Put in standard form.

Factor.

Use the Zero-Product Property and solve.

The original equation appears to have the solution set \{-1, 3\}. However, we have not yet checked.
Example 3: Solving a Radical Equation
(4 of 4)

Check:

$x = -1$

$$\sqrt{2x + 3} - \sqrt{x + 1} = \sqrt{2(-1) + 3} - \sqrt{(-1) + 1}$$
$$= \sqrt{1} - \sqrt{0} = 1 - 0 = 1$$

$x = 3$

$$\sqrt{2x + 3} - \sqrt{x + 1} = \sqrt{2(3) + 3} - \sqrt{3 + 1}$$
$$= \sqrt{9} - \sqrt{4} = 3 - 2 = 1$$

The solution set is \{-1, 3\}. 
Example 4: Solving an Equation by Factoring

Solve the equation: \( x^4 = 9x^2 \)

Begin by collecting all terms on one side. This results in 0 on the side and an expression to be factored on the other.

\[
x^4 - 9x^2 = 0
\]

\[
x^2(x^2 - 9) = 0
\]

\[
x^2 = 0 \text{ or } x^2 - 9 = 0
\]

\[
x = 0 \text{ or } x^2 = 9
\]

\[
x = 0 \text{ or } x = -3 \text{ or } x = 3
\]

Use the Zero-Product Property.

Use the Square Root Method.
Section 1.6
Equations and Inequalities Involving Absolute Value
Theorem

**THEOREM**

If $a$ is a positive real number and if $u$ is any algebraic expression, then $|u| = a$ is equivalent to $u = a$ or $u = -a$.
Example 1: Solving an Equation Involving Absolute Value (1 of 2)

Solve the equations:

a) \(|x + 5| = 15\)

Apply the Theorem, where \(u = x + 5\). There are two possibilities:

\[ x + 5 = 15 \quad \text{or} \quad x + 5 = -15 \]

\[ x = 10 \quad \text{or} \quad x = -20 \]

The solution set is \(\{-20, 10\}\).
Example 1: Solving an Equation Involving Absolute Value (2 of 2)

b) \( |1 - 2x| + 6 = 9 \)

Put the equation in the form of the Theorem by subtracting 6 from both sides of the equation.

\[
|1 - 2x| + 6 = 9 \\
|1 - 2x| = 3 \\
1 - 2x = 3 \quad \text{or} \quad 1 - 2x = -3 \\
-2x = 2 \quad \text{or} \quad -2x = -4 \\
x = -1 \quad \text{or} \quad x = 2
\]

The solution set is \( \{-1, 2\} \).
**Example 2: Solving an Inequality Involving Absolute Value**

Solve the inequality: $|x| < 3$

We are looking for all points whose coordinate $x$ is a distance less than 3 units from the origin.

Because any number $x$ between $-3$ and 3 satisfies the condition $|x| < 3$, the solution set consists of all numbers $x$ for which $-3 < x < 3$, that is, all real numbers in the interval $(-3, 3)$. 
Theorem

THEOREM
If $a$ is a positive number and if $u$ is an algebraic expression, then

- $|u| < a$ is equivalent to $-a < u < a$
- $|u| \leq a$ is equivalent to $-a \leq u \leq a$

In other words, $|u| < a$ is equivalent to $-a < u$ and $u < a$. 
Example 3: Solving an Inequality Involving Absolute Value (1 of 2)

Solve the inequality \(|2x + 3| \leq 6\), and graph the solution set.

\[
|2x + 3| \leq 6
\]

This follows the form of \(|u| \leq a\) where \(u = 2x + 3\).

- Use statement \(-a \leq u \leq a\).
- Subtract 3 from each part.
- Simplify.
- Divide each part by 2.
- Simplify.
Example 3: Solving an Inequality Involving Absolute Value (2 of 2)

The solution set is \( \left\{ x \mid -\frac{9}{2} \leq x \leq \frac{3}{2} \right\} \), that is, all real numbers in the interval \( \left[ -\frac{9}{2}, \frac{3}{2} \right] \).

The graph of the solution set is
Example 4: Solving an Inequality Involving Absolute Value (1 of 2)

Solve the inequality $|x| > 2$, and graph the solution set.

We are looking for all points whose coordinate $x$ is a distance greater than 2 units from the origin. The figure illustrates the situation.
Example 4: Solving an Inequality Involving Absolute Value (2 of 2)

Any number $x$ less than $-2$ or greater than $2$ satisfies the condition $|x| > 2$.

The solution set consists of all numbers $x$ for which $x < -2$ or $x > 2$, that is, all real numbers in $(-\infty, -2) \cup (2, \infty)$. 
Theorem

**THEOREM**

If $a$ is a positive number and $u$ is an algebraic expression, then

- $|u| > a$ is equivalent to $u < -a$ or $u > a$
- $|u| \geq a$ is equivalent to $u \leq -a$ or $u \geq a$

\[ |u| \geq a, \ a > 0 \]
Example 6: Solving an Inequality Involving Absolute Value (1 of 2)

Solve the inequality $|4x - 10| > 6$, and graph the solution set.

This follows the form of $|u| > a$ where $u = 4x - 10$.

$4x - 10 < -6$ or $4x - 10 > 6$

Use statement $u < -a$ or $u > a$.

Add 10 to each part.

$4x - 10 + 10 < -6 + 10$ or $4x - 10 + 10 > 6 + 10$

$4x < 4$ or $4x > 16$

Simplify.

$\frac{4x}{4} < \frac{4}{4}$ or $\frac{4x}{4} > \frac{16}{4}$

Divide each part by 4.

$x < 1$ or $x > 4$

Simplify.
Example 6: Solving an Inequality Involving Absolute Value (2 of 2)

The solution set is \( \{ x | x < 1 \text{ or } x > 4 \} \), that is, all real numbers in \((-\infty, 1) \cup (4, \infty)\).

The graph of the solution set is