MAT 1033

## Practice 19:

1) $x^2 = 49 \Rightarrow x = \pm\sqrt{49}$, $x = \pm 7$ or $x_1 = -7$, $x_2 = 7$

$\{-7, 7\}$

2) $p^2 = 20 \Rightarrow p = \pm\sqrt{20}$, $p = \pm\sqrt{4(5)}$, $p = \pm 2\sqrt{5}$

$\{-2\sqrt{5}, 2\sqrt{5}\}$

3) $(x-16)^2 = 9$

$x - 16 = \pm\sqrt{9}$

$x - 16 = \pm 3 \Rightarrow x = 16 \pm 3$

$x_1 = 16+3 = 19$

$x_2 = 16-3 = 13$

$\{13, 19\}$

4) $(4s+3)^2 = 4 \Rightarrow 4s+3 = \pm\sqrt{4}$

$4s + 3 = \pm 2$

$4s = -3 \pm 2$

$s = \dfrac{-3 \pm 2}{4}$

$s_1 = \dfrac{-3+2}{4} = \dfrac{-1}{4}$

$s_2 = \dfrac{-3-2}{4} = \dfrac{-5}{4}$

$\{-1/4, -5/4\}$

5) $x^2 = -49$

$x = \pm\sqrt{-49} \Rightarrow x = \pm\sqrt{49(i^2)}$

$\Rightarrow x = \pm 7i$

$\{-7i, 7i\}$

6) $(p+3)^2 = -6$ , $p+3 = \pm\sqrt{-6}$

$p+3 = \pm\sqrt{6(i^2)}$

$p+3 = \pm i\sqrt{6}$

$p = -3 \pm i\sqrt{6}$

$\{-3-i\sqrt{6}, -3+i\sqrt{6}\}$

7) $a^2 - 8a - 20 = 0$

$a^2 - 8a + \_\_\_ = 20$

$a^2 - 8a + \frac{4^2}{\div 2} = 20 + 4^2$

$(a-4)^2 = 36$ , $a-4 = \pm\sqrt{36}$

$a-4 = \pm 6$

$a = 4 \pm 6$

$a_1 = 4+6 = 10$

$a_2 = 4-6 = -2$

$\{10, -2\}$

8) $p^2 + 3p - 9 = 0$

$p^2 + 3p + \frac{(3/2)^2}{\div 2} = 9 + (3/2)^2$

$9 + \frac{9}{4} = \frac{45}{4}$

$\sqrt{\frac{45}{4}} = \frac{3\sqrt{5}}{2}$

$(p+3/2)^2 = \frac{45}{4}$ , $p+3/2 = \sqrt{\frac{45}{4}}$

$p+3/2 = \pm\frac{3\sqrt{5}}{2}$ $\therefore$ $p = -3/2 \pm \frac{3\sqrt{5}}{2}$

$p_1 = \frac{-3+3\sqrt{5}}{2}$ , $p_2 = \frac{-3-3\sqrt{5}}{2}$

$\frac{1}{3} \div 2 = 1/6$

$(1/3 \cdot 1/2 = 1/6)$

9) $9x^2 + 3x - 6 = 0$

$9(x^2 + \frac{1}{3}x) = 6$

$9(x^2 + 1/3 x + \underline{(1/6)^2}) = 6 + 9(1/36)$

$\div 2$

$9(x + 1/6)^2 = 6 + 1/4$,

$9(x+1/6)^2 = \frac{25}{4} \quad \therefore \quad (x+1/6)^2 = \frac{25}{36}$

$x + 1/6 = \pm\sqrt{\frac{25}{36}}$

$x + \frac{1}{6} = \pm \frac{5}{6} \quad \therefore \quad x = -\frac{1}{6} \pm \frac{5}{6}$

$x_1 = -1/6 + 5/6 = 4/6 = 2/3$

$x_2 = -1/6 - 5/6 = -6/6 = -1$

$\{-1, 2/3\}$

10. Quadratic formula: $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant $D = \sqrt{b^2 - 4ac}$

for $s^2 + 5s - 6 = 0$

$a = 1, \; b = 5, \; c = -6 \quad \therefore \quad D = \sqrt{25 - 4(1)(-6)}$

$D = \sqrt{49} = \pm 7$

two rational numbers

11) $t^2 - 2t + 1 = 0$

$a = 1, \quad b = -2, \quad c = 1$

$D = \sqrt{b^2 - 4ac}$

$D = \sqrt{(-2)^2 - 4(1)(1)}$

$D = \sqrt{4 - 4} = 0$

From $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

if $D = 0$, $x = -b/2a$. one solution.

12) $2y^2 + 3y + 5 = 0$ (rearrange $2y^2 = -3y - 5$)

$a = 2, \quad b = 3, \quad c = 5$

$D = \sqrt{b^2 - 4ac} = \sqrt{3^2 - 4(2)(5)} = \sqrt{9 - 40} = \sqrt{-31}$

$D = \pm i\sqrt{31}$ imaginary, complex.

two complex solutions.

13) $x^2 + x + 2 = 0$

$a = 1, \quad b = 1, \quad c = 2$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(2)}}{2(1)}$

$x = \dfrac{-1 \pm \sqrt{1 - 8}}{2}$

$x = \dfrac{-1 \pm \sqrt{-7}}{2}$

$x = \dfrac{-1 \pm i\sqrt{7}}{2}$

$x_1 = \dfrac{-1 + i\sqrt{7}}{2}$, $x_2 = \dfrac{-1 - i\sqrt{7}}{2}$

14) $6m^2 - 5m + 2 = 0$ $\quad a = 6, \quad b = -5, \quad c = 2$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(6)(2)}}{2(6)} = \frac{5 \pm \sqrt{25 - 48}}{12}$$

$$x = \frac{5 \pm \sqrt{-23}}{12}$$

$$x_1 = \frac{5 + i\sqrt{23}}{12}, \qquad x_2 = \frac{5 - i\sqrt{23}}{12}.$$

15) $3n^2 + 12n + 5 = 0$, $\quad a = 3, \quad b = 12, \quad c = 5$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(3)(5)}}{2(3)} = \frac{-12 \pm \sqrt{144 - 60}}{6}$$

$$x = \frac{-12 \pm \sqrt{84}}{6} = \frac{-12 \pm \sqrt{4(21)}}{6} = \frac{-12 \pm 2\sqrt{21}}{6}$$

$$x = \frac{2(-6 \pm \sqrt{21})}{6}, \qquad x = \frac{-6 \pm \sqrt{21}}{3}$$

$$x_1 = \frac{-6 + \sqrt{21}}{3}, \quad x_2 = \frac{-6 - \sqrt{21}}{3}$$

16) $x^2 + x + 3 = 0$

$a = 1, \quad b = 1, \quad c = 3$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(3)}}{2(1)} = \frac{-1 \pm \sqrt{-11}}{2} = \frac{-1 \pm i\sqrt{11}}{2}$$

$$x = \frac{-1 + i\sqrt{11}}{2}, \qquad x = \frac{-1 - i\sqrt{11}}{2}$$