

MAP2302: Differential Equations.

<http://www.imathesis.com/map2302.html>

Review Test 4. Topics: Annihilator method. Series Solutions to Differential Equations.

Find a linear differential operator that annihilates the given function:

a) $1 + x^3 + e^{2x} \sin 3x$ b) $2 + x^2 \cos x$ c) $xe^{-x} \sin x - 2e^x \cos 2x$

Find a general solution to the given ODE using the annihilator method:

1. $y'' + 4y' + 4y = 2x + 6$

2. $y'' + 3y' = 4x - 5$

3. $y'' - y' = x \cos 2x$

4. $y''' + y'' = 8x^2 e^x$

5. $y'' - 2y' - 3y = 4e^x \sin x$

6. $y'' + 25y = 6 \sin x$

7. $y'' + 6y' + 9y = -xe^{4x}$

8. $y'' - y = x^2 e^x + 2 \cos x$

9. $y'' - 2y' + 5y = xe^x \sin 2x$

Solve the Initial Value problem:

a) $y'' - 4y' + 8y = x^3$, $y(0) = 2$, $y'(0) = 4$

b) $y^{(4)} - y''' = x + e^x$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 0$, $y'''(0) = 0$

Answers to IVP:

a) $y = 2e^{2x} \cos 2x - \frac{3}{64}e^{2x} \sin 2x + \frac{1}{8}x^3 + \frac{3}{16}x^2 + \frac{3}{32}x$

b) $y = 2 + x - 2e^x - \frac{1}{6}x^3 - \frac{1}{24}x^4 + xe^x$

Find at least the first three nonzero terms in a power series expansion about $x = 0$ for a general solution to the given differential equation:

a) $y'' - xy' + 4y = 0$ Ans: $y(x) = a_0 \left(1 - 2x^2 + \frac{1}{3}x^4 + \dots\right) + a_1 \left(x - \frac{1}{2}x^3 + \frac{1}{40}x^5 + \dots\right)$

Find at least the first four nonzero terms in a power series expansion about $x = 0$ for the solution to the given initial value problem:

a) $y'' + 3xy' - y = 0$, $y(0) = 2$, $y'(0) = 0$.

Ans: $y(x) = 2 + x^2 - \frac{5}{12}x^4 + \frac{11}{72}x^6 + \dots$