MAP2302: Differential Equations.

http://www.imathesis.com/map2302.html

Review Test 3.

Topics: The Laplace Transform.

Practices 10 and 11 should be considered part of the preparation for Test 3.

1. Find the Laplace Transform of the following functions:

a) $f(t) = 4t^2 - 2t + 3$ b) $f(t) = 3\sin 5t - 2\cos 3t$ c) $f(t) = 3e^{2t} + 5e^{-3t}$

d) $f(t) = e^{-2t}(4\cos 5t + 3\sin 5t)$ e) $f(t) = t^3e^{2t} + 2te^{-t}$ f) $(1 + e^{3t})^2$

2. Use the appropriate trigonometric identity to find the following Laplace Transforms:

a) $\mathcal{L}\{\cos^2 t\}$ b) $\mathcal{L}\{\sin 2t \cos 2t\}$ c) $\mathcal{L}\{\sin 3t \cos 4t\}$

3. Use the method of partial fractions to find the given inverse Laplace transforms:

a) $\mathcal{L}^{-1}\Big\{\dfrac{s+3}{s^2+4s-5}\Big\}$ b) $\mathcal{L}^{-1}\Big\{\dfrac{s+1}{(2s-1)(s+2)}\Big\}$ c) $\mathcal{L}^{-1}\Big\{\dfrac{s^2+s+5}{s^4-s^2}\Big\}$

4. Use Laplace Transforms to solve the following initial-value problems:

i. $y' + 4y = e^t;\ \ y(0) = 2$

ii. $y' - y = \sin t;\ \ y(0) = 1$

iii. $y'' + 3y' + 2y = t + 1;\quad y(0) = 1,\ \ y'(0) = 0$

iv. $y'' + 4y' + 13y = 0;\quad y(0) = 1,\ \ y'(0) = 2$

v. $y'' + 4y = 4t + 8;\quad y(0) = 4,\ \ y'(0) = -1$

vi. $y'' + y' - 2y = 5e^{3t};\quad y(0) = 1,\ \ y'(0) = -4$

vii. $y'' + y' - 2y = e^t;\quad y(0) = 2,\ \ y'(0) = 3$

viii. $y'' - 2y' + y = e^t;\quad y(0) = 3,\ \ y'(0) = 4$

ix. $y'' + 2y' + 2y = \cos 2t;\quad y(0) = 0,\ \ y'(0) = 1$

x. $y'' + 4y = \sin 3t;\quad y(0) = 2,\ \ y'(0) = 1$

xi. $y'' + 2y' + 5y = 3e^{-t}\cos 2t;\quad y(0) = 1,\ \ y'(0) = 2$

Answers on the back page:

Answers:

1.

a) $\frac{8}{s^3} - \frac{2}{s^2} + \frac{3}{s}$; b) $\frac{15}{s^2+25} - \frac{2s}{s^2+9}$; c) $\frac{3}{s-2} + \frac{5}{s+3}$;

d) $\frac{4(s+2)}{(s+2)^2+25} + \frac{15}{(s+2)^2+25}$; e) $\frac{6}{(s-2)^4} + \frac{2}{(s+1)^2}$; f) $\frac{1}{s} + \frac{2}{s-3} + \frac{1}{s-6}$

2.

a) $\frac{1}{2}\left(\frac{1}{s} + \frac{s}{s^2+4}\right)$; b) $\left(\frac{2}{s^2+16}\right)$; c) $\frac{1}{2}\left(\frac{7}{s^2+49} - \frac{1}{s^2+1}\right)$.

3.

a) $\frac{2}{3}e^t + \frac{1}{3}e^{-5t}$ b) $\frac{3}{10}e^{t/2} + \frac{1}{5}e^{-2t}$ c) $-1 - 5t + \frac{7}{2}e^t - \frac{5}{2}e^{-t}$

4. Initial value problems:

i. $y(t) = \frac{9}{5}e^{-4t} + \frac{1}{5}e^t$

ii. $y(t) = \frac{3}{2}e^t - \frac{1}{2}\cos t - \frac{1}{2}\sin t$

iii. $y(t) = \frac{1}{2}t - \frac{1}{4} + 2e^{-t} - \frac{3}{4}e^{-2t}$

iv. $y(t) = e^{-2t}\cos 3t + \frac{4}{3}e^{-2t}\sin 3t$

v. $y(t) = t + 2 + \cos 2t - \sin 2t$

vi. $y(t) = 2e^{-2t} - \frac{3}{2}e^t + \frac{1}{2}e^{3t}$

vii. $y(t) = -\frac{2}{9}e^{-2t} + \frac{20}{9}e^t + \frac{1}{3}te^t$

viii. $y(t) = 3e^t + te^t + \frac{1}{2}t^2e^t$

ix. $y(t) = \frac{1}{10}e^{-t}\cos t + \frac{7}{10}e^{-t}\sin t - \frac{1}{10}\cos 2t + \frac{1}{5}\sin 2t$

x. $y(t) = 2\cos 2t + \frac{4}{5}\sin 2t - \frac{1}{5}\sin 3t$

xi. $y(t) = e^{-t}\cos(2t) + \frac{3}{2}e^{-t}\sin(2t) + \frac{3}{4}te^{-t}\sin(2t)$