

## MAP2302: Differential Equations.

<http://www.imathesis.com/map2302.html>

### Test 2 Review

1. Solve the following homogeneous DEs:

a.  $y'' - 10y' + 25y = 0$                       Ans:  $y(t) = C_1e^{5t} + C_2te^{5t}$

b.  $y''' - 6y'' + 11y' - 6y = 0$                       Ans:  $y(x) = C_1e^x + C_2e^{2x} + C_3e^{3x}$

2. Solve the following initial value problem:

a.  $4y'' + 4y' + 17y = 0$ ,  $y(0) = -1$ ;  $y'(0) = 2$                       Ans:  $y(x) = e^{-\frac{x}{2}} \left( -\cos 2x + \frac{3}{4}\sin 2x \right)$

3. Find a general solution for the following DEs using the method of undetermined coefficients.:

a.  $y'' - y' - 2y = e^{3x}$                       Ans:  $y(x) = C_1e^{-x} + C_2e^{2x} + \frac{1}{4}e^{3x}$

b.  $y'' + 3y' + 2y = \sin t$                       Ans:  $y(t) = C_1e^{-t} + C_2e^{-2t} + 0.1\sin t - 0.3\cos t$

c.  $y'' + 4y = 5t^2e^t$                       Ans:  $y(t) = C_1\cos 2t + C_2\sin 2t + \left( t^2 - \frac{4}{3}t - \frac{2}{25} \right) e^t$

d.  $y'' + 2y' - 3y = 7\cos(3t)$                       Ans:  $y(t) = C_1e^t + C_2e^{-3t} - \frac{7}{15}\cos 3t + \frac{7}{30}\sin 3t$

4. Calculate the Wronskian determinant and decide whether the functions  $y_1$  and  $y_2$  are linearly dependent:

a.  $y_1(t) = e^{3t}$ ,  $y_2(t) = e^{-4t}$                       Ans:  $W = -7e^{-t} \neq 0$ ; therefore, linearly independent.

b.  $y_1(t) = te^{2t}$ ,  $y_2(t) = e^{2t}$                       Ans:  $W = -e^{4t} \neq 0$ ; therefore, linearly independent.

5. Find the equation of motion for the vibrating spring with damping if  $m = 2 \text{ kg}$ ,  $b = 12 \text{ kg/sec}$ ,  $k = 50 \text{ kg/sec}^2$ ,  $y(0) = 0.5 \text{ m}$  and  $y'(0) = -0.2 \text{ m/sec}$ .

(b) After how many seconds will the mass first cross the equilibrium point?

Ans:              Equation:  $y(t) = e^{-3t} (0.5\cos 4t + 0.325\sin 4t)$     (a)  $t \approx 0.54 \text{ secs}$

6. Find a general solution to the differential equation using the method of variation of parameters:

a.  $y'' + y = \sec(x)$                       Ans:  $y(x) = C_1\cos x + C_2\sin x + x\sin x + \cos x \ln|\cos x|$

b.  $y'' - 2y' + y = \frac{e^x}{x}$                       Ans:  $y(x) = C_1e^x + C_2xe^x + xe^x\ln|x|$

c.  $y'' - y' - 2y = e^{3x}$                       Ans:  $y(x) = C_1e^{-x} + C_2e^{2x} + \frac{1}{4}e^{3x}$

7. For the given Euler-Cauchy equations, solve the IVP:

a.  $x^2y'' + 3xy' = 0$ ,  $y(1) = 0$ ,  $y'(1) = 4$                       Ans:  $y(x) = 2 - 2x^{-2}$

b.  $x^2y'' - 5xy' + 8y = 0$ ,  $y(2) = 32$ ,  $y'(2) = 0$                       Ans:  $y(x) = 16x^2 - 2x^4$

c.  $x^2y'' + xy' + y = 0$ ,  $y(1) = 1$ ,  $y'(1) = 2$                       Ans:  $y(x) = \cos(\ln x) + 2\sin(\ln x)$