MAP2302: Differential Equations.

http://www.imathesis.com/map2302.html

Review Final Exam.

1. Classify the following ODEs by order and as linear or non-linear:

a) $e^y \dfrac{d^2y}{dx^2} + 2\left(\dfrac{dy}{dx}\right)^2 = 1$

b) $y'' + yy' + xy = 0$

c) $x^2y'' + e^x y = x.$

2. Consider the initial-value problem $y' = 0.1\sqrt{y} + 0.4x^2, \;\; y(2) = 4.$ Use Euler's method to obtain an approximation of $y(2.5)$ using $h = 0.1$

3. Solve the equation by separation of variables:

a) $x\dfrac{dv}{dx} = \dfrac{1-4v^2}{3v}$

4. Solve First order Linear equations by using the appropriate integrating factor.

a) $\dfrac{dy}{dx} + y\tan x = \sec x$

b) $x\dfrac{dy}{dx} + 2y = 5x^3$

5. Solve the following *exact* ODE:

a) $(2xy+3)\,dx + \left(x^2-1\right)dy = 0$

6. Solve the following ODE by using the substitution $v = \dfrac{y}{x}$ or $v = \dfrac{x}{t}$:

a) $(xy+y^2)dx - x^2dy = 0$

7. Use the method discussed under *Bernoulli Equations* to solve the following ODE:

a) $\dfrac{dy}{dx} + \dfrac{y}{x} = x^2y^2$

8. Solve the initial value problem:

a. $y'' + 2y' - 8y = 0; \quad y(0) = 3, \; y'(0) = -12$

9. Solve the given initial value problem:

a. $y'' + 2y' + 2y = 0; \; y(0) = 2, \; y'(0) = 1$

10. Find a particular solution to the differential equation using the Method of Undetermined Coefficients:

a) $2x' + x = 3t^2$

Note for question 11: Solution of the form: $y_p(t) = v_1 y_1 + v_2 y_2$ where v_1 and v_2 are functions of the independent variable.

$$v_1 = \int \frac{-y_2 \cdot f(t)}{W}$$
$$v_2 = \int \frac{y_1 \cdot f(t)}{W}$$

11. Find a general solution to the differential equation using the method of variation of parameters:

a) $y'' + 4y = \tan(2t)$

b) $y'' - 2y' + y = t^{-1}e^t$

12. Find a general solution to the given Cauchy-Euler equation for $t > 0$:

a) $t^2 y'' + 7ty' - 7y = 0$

b) $t^2 z'' + 5tz' + 4z = 0$

13. Determine the Laplace Transform of the given function using a Table of Transforms and the properties of the Transform:

a) $t^2 + e^t \sin(2t)$

b) $e^{-t}\cos(3t) + e^{6t} - 1$

Solve the initial value problem using the method of Laplace Transforms:

14. $y'' - 2y' + 5y = 0; \quad y(0) = 2, \quad y'(0) = 4$

15. Use the annihilator method to determine the form of a general solution for the given equation:

a) $y'' - 5y' + 6y = \cos 2x + 1$

15. Find at least the first four nonzero terms in a power series expansion about $x = 0$ for a general solution to the given differential equation:

a) $y' + (x + 2)y = 0$