MAP2302: Differential Equations.

http://www.imathesis.com/map2302.html

Practice 12.

Topics: 6.2;, 6.3: Homogeneous Linear Equations with constant coefficients. The Annihilator method.

Note: Finding the annihilator follow these very informal rules:

1. For separated terms, an annihilator multiplies the other;
2. In different terms, but one includes the other –one is a factor of the other, take the one at the highest power;
3. Within the same term, one goes into the other.

Basic annihilators: D^{n+1} annihilates x^n; $(D-\alpha)$ annihilates $e^{\alpha x}$ and $\left(D^2+\beta^2\right)$ annihilates $\sin\beta x$ or $\cos\beta x$.

Exercises 6.3 page 337: Find a differential operator that annihilates the given function:

11. x^4-x^2+11 12. $3x^2-6x+1$ Ans (12): D^3

13. e^{-7x} 14. e^{5x} Ans(14): $(D-5)$

15. $e^{2x}-6e^x$ 16. x^2-e^x Ans(16): $D^3(D-1)$

17. $x^2e^{-x}\sin 2x$ 18. $xe^{3x}\cos 5x$ Ans(18): $\left[(D-3)^2+25\right]^2$

19. $xe^{-2x}+xe^{-5x}\sin 3x$ 20. $x^2e^x-x\sin 4x+x^3$ Ans(20): $D^4(D-1)^3(D^2+16)^2$

Examples taken from another book (Differential equations by Dennis Zill: Exercises 4.5 page 156, ninth edition):

15. $1+6x-2x^3$ Ans: D^4

16. $x^3(1-5x)$ Ans: D^5

17. $1+7e^{2x}$ Ans: $D(D-2)$

18. $x+3xe^{6x}$ Ans: $D^2(D-6)^2$

19. $\cos 2x$ Ans: D^2+4

20. $1+\sin x$ Ans: $D(D^2+1)$

21. $13x+9x^2-\sin 4x$ Ans: $D^3(D^2+16)$

22. $8x-\sin x+10\cos 5x$ Ans: $D^2(D^2+1)(D^2+25)$

23. $e^{-x}+2xe^x-x^2e^x$ Ans: $(D+1)(D-1)^3$

25. $3+e^x\cos 2x$ Ans: $D\left[(D-1)^2+4\right]=D[D^2+2D+5]$

26. $e^{-x}\sin x-e^{2x}\cos x$ Ans: $[(D+1)^2+1][(D-2)^2+1]=(D^2+2D+2)(D^2-2D+5]$