

MAP2302: Differential Equations.

<http://www.imathesis.com/map2302.html>

Practice 9.

Topics: 4.7 Variable-Coefficient Equations.

Note: Cauchy-Euler equation: $at^2y'' + bty' + cy = f(t)$

Try t^r , which yields auxiliary equation: $ar(r-1) + br + c = 0$; three cases:

1. Distinct real roots: $y = C_1t^{r_1} + C_2t^{r_2}$
2. Repeated real roots: $y = C_1t^{r_1} + C_2t^{r_2}\ln t$
3. Complex roots: $a \pm bi$: $y = t^a[C_1\cos(b\ln t) + C_2\sin(b\ln t)]$

Examples to discuss in class:

1. Distinct real roots: $x^2y'' - 2xy' - 4y = 0$
2. Repeated real roots: $4x^2y'' + 8xy' + y = 0$
3. Complex roots: $4x^2y'' + 17y = 0$

Exercises 4.7 page 199:

Find a general solution to the given Cauchy-Euler equation for $t > 0$:

9. $t^2y'' + 7ty' - 7y = 0$

11. $t^2z'' + 5tz' + 4z = 0$

13. $9t^2y'' + 15ty' + y = 0$

Find a general solution for $t < 0$:

15. $y'' - \frac{1}{t}y' + \frac{5}{t^2}y = 0$

17. $t^2y'' + 9ty' + 17y = 0$

Solve the initial value problem:

19. $t^2y'' - 4ty' + 4y = 0$

$y(1) = -2, y'(1) = -11$

Devise a modification of the method to find a general solution to the given equation:

21. $(t-2)^2y'' - 7(t-2)y' + 7y = 0, t > 0.$