MAP2302: Differential Equations.

http://www.imathesis.com/map2302.html

Practice 7.

Topics: 4.4 & 4.5 Nonhomogeneous Equations: the Method of Undetermined Coefficients.

Terminology: Characteristic polynomial, auxiliary equation, complementary solution, null solution, particular solution, general solution. The superposition principle.

Examples:

a. $y'' + 4y' - 2y = 2x^2 - 3x + 6$; Ans: $y_p = -x^2 - \frac{5}{2}x - 9$.

b. $y'' - y' + y = 2\,\sin(3x)$; Ans: $y_p = \frac{6}{73}\cos(3x) - \frac{16}{73}\sin(3x)$

c. $y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$; Ans: $y_p = -\frac{4}{3}x + \frac{23}{9} - \left(2x + \frac{4}{3}\right)e^{2x}$

d. $y'' - 6y' + 9y = e^{3x}$; Ans: General Solution: $y(x) = C_1e^{3x} + C_2xe^{3x} + \frac{1}{2}x^2e^{3x}$

1. Find a particular solution to the differential equation (problems taken from 9 to 26, p180):

9. $y'' + 3y = -9$

11. $y'' + y = 2^x$

12. $2x' + x = 3t^2$ Ans: $x_p(t) = 3t^2 - 12t + 24$

13. $y'' - y' + 9y = 3\,\sin(3t)$

14. $2z'' + z = 9e^{2t}$ Ans: $z_p(t) = e^{2t}$

15. $y'' - 5y' + 6y = xe^x$

17. $y'' + 4y = 8\,\sin(2t)$

23. $y''(\theta) - 7y'(\theta) = \theta^2$

24. $y'' + y = 4x\,\cos(x)$ Ans: $y_p(x) = x\,\cos(x) + x^2\,\sin(x)$

From exercises 4.5 page 185:

1. Given that $y_1(t) = \cos(t)$ is a solution to $y'' - y' + y = \sin(t)$ and $y_2(t) = e^{2t}/3$ is a solution to $y'' - y' + y = e^{2t}$ use the superposition principle to find solutions to the following DEs:

a. $y'' - y' + y = 5\,\sin(t)$
b. $y'' - y' + y = \sin(t) - 3e^{2t}$
c. $y'' - y' + y = 4\sin(t) + 18e^{2t}$

From exercises 4.5 page 186: Find a general solution to the DE:

17. $y'' - 2y' - 3y = 3t^2 - 5$

19. $y'' - 3y' + 2y = e^x\sin(x)$

A vibrating spring with damping that is under external force can be modeled by $my + by + ky = g(t)$ where m is the mass of the spring system, b is the damping constant, k is the spring constant, $g(t)$ is the force on the system at time t, and $y(t)$ is the displacement from the equilibrium of the spring system at time t.

43. A mass spring system is driven by a sinusoidal external force $g(t) = 5\sin(t)$. The mass equals 1, the spring constant equals 3 and the damping coefficient equals 4. Find its equation of motion.

Ans: $y(t) = -\cos(t) + \frac{1}{2}\sin(t) + C_1e^{-3t} + C_2e^{-t}$