**MAP2302: Differential Equations.**

*http://www.imathesis.com/map2302.html*

Practice 6.

**Topics**: 4.3 Linear Second Order Homogeneous Equations with Complex Roots.

1. The auxiliary equation for the given differential equation has complex roots ( a through d ). For e through h, find a general solution (odd problems p.172):

a. $y'' + 9y = 0$
b. $z'' - 6z' + 10z = 0$
c. $w'' + 4w' + 6w = 0$
d. $4y'' + 4y' + 6y = 0$
e. $y'' - 8y' + 7y = 0$
f. $z'' + 10z' + 25z = 0$
g. $y'' - 2y' + 26y = 0$
f. $y'' - 3y' - 11y = 0$
g. $y'' - y' + 7y = 0$
h. $y''' + y'' + 3y' - 5y = 0$

2. Solve the given initial value problem:

a. $y'' + 2y' + 2y = 0;\ y(0) = 2,\ y'(0) = 1$
b. $w'' - 4w' + w = 0;\ w(0) = 0,\ w'(0) = 1$
c. $y'' - 2y' + 2y = 0;\ y(\pi) = e^{\pi},\ y'(\pi) = 0$
d. $y''' - 4y'' + 7y' - 6y = 0;\ y(0) = 1,\ y'(0) = 0,\ y''(0) = 0$

3. Find a general solution to the following higher-order equations:

a. $y''' - y'' + y' + 3y = 0$
b. $y''' + 2y'' + 5y' - 26y = 0$
c. $y^{iv} + 13y'' + 36y = 0$

32. Page 173. **Vibrating Spring without Damping**. A vibrating spring without damping can be modeled by the initial value problem $my''(t) + by'(t) + ky(t) = 0$ by taking $b = 0$.
(a) If $m = 10\ kg,\ \ k = 250\ kg/sec^2,\ \ y'(0) = 0.3\ m$ and $y(0) = -0.1\ m/sec$, find the equation of motion for this undamped vibrating spring.
(b) After how many seconds will the mass in part (a) first cross the equilibrium point?
(c) Find the Period and the Frequency of the oscillation.

33. Page 173. **Vibrating Spring with Damping**.
(a) Find the equation of motion for the vibrating spring with damping if $m = 10\ kg,\ b = 60\ kg/sec,\ k = 250\ kg/sec^2$, $y(0) = 0.3\ m$ and $y(0) = -0.1\ m/sec$.
(b) After how many seconds will the mass in part (a) first cross the equilibrium point?
(c) Find the frequency of oscillation for the spring system.
(d) Compare the results of Problems 32 and 33 and determine what effect the damping has on the frequency of oscillation.