MAP2302: Differential Equations.

http://www.imathesis.com/map2302.html

Practice 4

Topics: 2.4 Exact Equations, page 64. 2.6 Substitutions and Transformations. Bernoulli Equations. Page 76.

1. Show that the following equation is not exact: $\left(\frac{1}{y}\right) dx - \left(3y - \frac{x}{y^2}\right) dy = 0$

2. Solve the following *exact* ODEs:

a) $(2xy+3)\,dx + \left(x^2-1\right) dy = 0$

b) $\left(e^x \sin y - 3x^2\right) dx + \left(e^x \cos y + y^{-\frac{2}{3}}/3\right) dy = 0$

c) $e^t(y-t)dt + (1+e^t)dy = 0$

d) $\cos\theta dr - (r\sin\theta - e^\theta)d\theta = 0$

3. Solve the initial value problem:

a) $\left(\frac{1}{x} + 2y^2x\right) dx + (2yx^2 - \cos y)dy = 0, \quad y(1) = \pi.$

b) $(e^t y + te^t y)dt + (te^t + 2)dy = 0, \quad y(0) = -1$

4. Solve the following ODEs by using the substitution $v = \frac{y}{x}$ or $v = \frac{x}{t}$:

a) $(xy+y^2)dx - x^2dy = 0$

b) $(y^2 - xy)dx + x^2dy = 0$

c) $\frac{dx}{dt} = \frac{x^2 + t\sqrt{t^2+x^2}}{tx}$

d) $\frac{dy}{dx} = \frac{x^2 - y^2}{3xy}$

5. Use the method discussed under *Bernoulli Equations* to solve the following ODEs:

a) $\frac{dy}{dx} + \frac{y}{x} = x^2y^2$

b) $\frac{dy}{dx} = \frac{2y}{x} - x^2y^2$

c) $\frac{dx}{dt} + tx^3 + \frac{x}{t} = 0$

d) $\frac{dr}{d\theta} = \frac{r^2 + 2r\theta}{\theta^2}$

Another mixing problem taken from textbook, page 93, slightly modified:

Consider a large tank holding 1000 L of pure water into which a brine solution of salt begins to flow at a constant rate of 6 L/min. The solution inside the tank is kept well stirred and is flowing out of the tank at a rate of 6 L/min. If the concentration of salt in the brine entering the tank is 0.1 kg/L, determine the mass of salt in the tank after 20 mins. (see Figure 3.2).

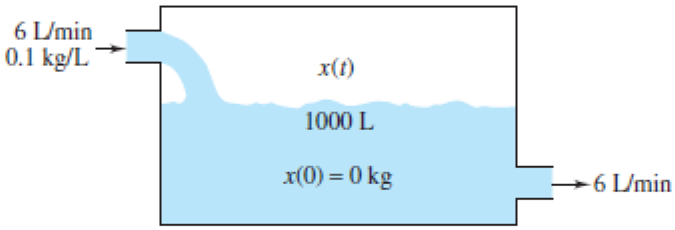

Figure 3.2 Mixing problem with equal flow rates

ANS: Eqn: $x(t) = 100(1 - e^{-3t/500})$; mass after 20 mins = 11.3 kg.

Newton Law of cooling word problem (Problem 1, page 107):

A cup of hot coffee initially at $95°C$ cools to $80°C$ in 5 min while sitting in a room of temperature $21°C$. Using just Newtons law of cooling, determine when the temperature of the coffee will be a nice $50°C$. Solve the problem by using the differential equation:

$$\frac{dT}{dt} = k(M - T)$$

ANS: Eqn: $T = 21 + 74e^{-0.0453t}$; time at which the coffee cup reaches $50°C$ is 20.7 mins.