MAP2302: Differential Equations.

http://www.imathesis.com/map2302.html

Practice 3

Topics: 2.2 Separable Equations. 2.3 Linear First Order Equations

1. Solve the equations by separation of variables (odd problems 7-16, page 46):

a) $x\dfrac{dy}{dx} = \dfrac{1}{y^3}$

b) $\dfrac{dx}{dt} = \dfrac{t}{xe^{t+2x}}$

c) $x\dfrac{dv}{dx} = \dfrac{1-4v^2}{3v}$

d) $\dfrac{dy}{dx} = 3x^2(1+y^2)^{\frac{3}{2}}$

e) $(x+xy^2)dx + e^{x^2}ydy = 0$

1.1 Solve the IVP by separation of variables (odds problems 17-26):

a) $\dfrac{dy}{dx} = (1+y^2)\tan x,\ \ y(0) = \sqrt{3}$

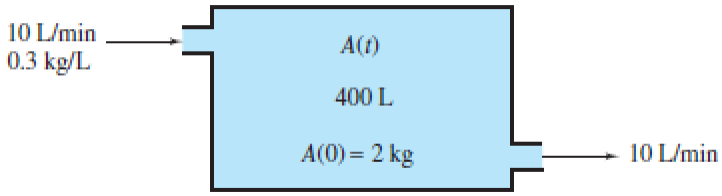
b) $\dfrac{1}{2}\dfrac{dy}{dx} = \sqrt{y+1}\,\cos x,\ \ y(\pi) = 0$

c) $\dfrac{1}{\theta}\dfrac{dy}{d\theta} = \dfrac{y\,\sin\theta}{y^2+1},\ \ y(\pi) = 1$

d) $\dfrac{dy}{dt} = 2t\,\cos^2 y,\ \ y(0) = \dfrac{\pi}{4}$

e) $\dfrac{dy}{dx} = x^2(1+y),\ y(0) = 3.$

Problem 33 page 47. Mixing. Suppose a brine containing 0.3 kilogram (kg) of salt per liter (L) runs into a tank initially filled with 400 L of water containing 2 kg of salt. If the brine enters at 10 L/min, the mixture is kept uniform by stirring, and the mixture flows out at the same rate. Find the mass of salt in the tank after 10 min (see Figure). Hint: Let A denote the number of kilograms of salt in the tank at t min after the process begins and use the fact that: *rate of increase in* A = *rate of input - rate of exit.*



Problem 34, page 47: Newtons Law of Cooling. According to Newtons law of cooling, if an object at temperature T is immersed in a medium having the constant temperature M, then the rate of change of T is proportional to the difference of temperature M T. This gives the differential equation:

$$\frac{dT}{dt} = k(M - T)$$

(a) Solve the differential equation for T.

(b) A thermometer reading $100°F$ is placed in a medium having a constant temperature of $70°F$. After 6 min, the thermometer reads $80°F$. What is the reading after 20 min?

ANS to #34: a) $T = M + 30e^{-kt}$ b) $70.77°F$

2.3. Solve First order Linear equations by using the appropriate integrating factor. (Odd Problems 7 to 22, page 54):

a) $\frac{dy}{dx} - y - e^{3x} = 0$

b) $\frac{dr}{d\theta} + r\tan\theta = \sec\theta$

c) $y\frac{dx}{dy} + 2x = 5y^3$

d) $(x^2 + 1)\frac{dy}{dx} + xy - x = 0$

Initial Value Problems:

a) $\frac{dy}{dx} - \frac{y}{x} = xe^x, \;\; y(1) = e - 1$

b) $t^2\frac{dx}{dt} + 3tx = t^4\ln t + 1, \;\; x(1) = 0$

c) $\cos x\frac{dy}{dx} + y\sin x = 2x\cos^2 x, \;\; y\left(\frac{\pi}{4}\right) = \frac{-15\sqrt{2}\;\pi^2}{32}$