MAP2302: Differential Equations.

http://www.imathesis.com/map2302.html

Practice 1

Topics: Intro. ODE. PDE. Order. Linear and non-linear ODE. General Solution. Particular Solution. Explicit and Implicit Solutions. Initial Value Problems: First and Second Order IVP. Existence and Uniqueness of Solution. Slope Fields.

I. Questions to be discussed in class:

1. Classify the following ODEs as linear or non linear and state the order:

a) $(1-x)y'' - 4xy' + 5y = \cos x$

b) $x\dfrac{d^3y}{dx^3} - \left(\dfrac{dy}{dx}\right)^4 + y = 0$

c) $t^5 y^{(4)} - t^3 y'' + 6y = 0$

d) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + y = \cos(x+y)$

e) $x'' + 16x = 0$

f) $\dfrac{dy}{dx} = -\dfrac{x}{y}$

g) $y'' - 2y' + y = 0$

h) $(1-y)y' + 2y = e^x$

i) $y'' + \sin y = 0$.

2. Show that $y = x^2 - x^{-1}$ is an explicit solution to the linear equation $y'' - \dfrac{2}{x^2}y = 0$.

3. Show that the relation $y^2 - x^3 + 8 = 0$ implicitly defines a solution to the nonlinear equation $y' = \dfrac{3x^2}{2y}$.

4. Verify that for every constant C the relation $4x^2 - y^2 = C$ is an implicit solution to $y\dfrac{dy}{dx} - 4x = 0$.

5. Show that $y = \sin x - \cos x$ is a solution to the initial value problem:

$y'' + y = 0; \quad y(0) = -1 \quad y'(0) = 1.$

6. Use the *Existence and Uniqueness of Solution* Theorem to show that $3y' = x^2 - xy^3$, $y(1) = 6$ has a unique solution.

7. Use the *Existence and Uniqueness of Solution* Theorem to show that $y' = 3y^{\frac{2}{3}}$, $y(2) = 0$ does not imply the existence of a unique solution.

8. Draw the isoclines with their direction markers and sketch several solution curves including the curve satisfying the given initial conditions:

a) $y' = -\dfrac{x}{y}, \quad y(0) = 4$ b) $y' = y, \quad y(0) = 1$ c) $y' = x, \quad y(0) = 1$.

II. Additional questions:

1. Classify the following ODEs by order and as linear or non-linear:

a) $e^y \dfrac{d^2y}{dx^2} + 2\left(\dfrac{dy}{dx}\right)^2 = 1$

b) $y'' + yy' + xy = 0$

c) $x^2y'' + e^x y = x$.

2. Is $y(x) = c_1\sin 2x + c_2\cos 2x$, where c_1 and c_2 are arbitrary constants, a solution of $y'' + 4y = 0$?

3. Determine whether $y = x^2 - 1$ is a solution of $(y')^4 + y^2 = -1$?

4. Classify the given problems (do not solve) as Initial Value Problem or Boundary-Value Problem. Explain the difference between the two.

a) $y'' + 2y' = e^x$ $y(0) = 1$ and $y(1) = 1$.
b) $y'' + 2y' = e^x$ $y(\pi) = 1$ and $y'(\pi) = 2$ and $y(\pi) = 1$.

5. Determine whether $y(x) = 2e^{-x} + xe^{-x}$ is a solution of $y'' + 2y' + y = 0$.

6. Show that $y = \ln x$ is a solution of $xy'' + y' = 0$ on Interval $(0, \infty)$ but is not a solution on the Interval $(-\infty, \infty)$

7. Find the solution to the IVP $y' + y = 0$; $y(3) = 2$ if the general solution is known to be $y(x) = c_1e^{-x}$.

8. Find the solution to the IVP $y'' + 4y = 0$; $y(0) = 0$ and $y'(0) = 1$.
if the general solution to the ODE is known to be $y(x) = c_1\sin 2x + c_2\cos 2x$.

9. Which of the following functions are solutions of the differential equation $y' - 5y = 0$? Justify.

a) $y = 5$ b) $y = 5x$ c) $y = x^5$ d) $y = e^{5x}$ e) $y = 2e^{5x}$ f) $y = 5e^{2x}$

10. Find the values of c_1 and c_2 so that the given functions will satisfy the prescribed initial conditions:

a) $y(x) = c_1e^x + c_2e^{-x} + 4\sin x$; $y(0) = 1$; $y'(0) = -1$;

b) $y(x) = c_1x + c_2 + x^2 - 1$ $y(1) = 1$; $y'(1) = 2$;

c) $y(x) = c_1e^x + c_2e^{2x} + 3e^{3x}$ $y(0) = 0$; $y'(0) = 0$;

d) $y(x) = c_1\sin x + c_2\cos x + 1$; $y(\pi) = 0$; $y'(\pi) = 0$;

e) $y(x) = c_1e^x + c_2xe^x + x^2e^x$ $y(1) = 1$; $y'(1) = -1$;