

# Review 4. IVP (b):

Solve IVP:

$$(4) \quad y^{(4)} - y''' = x + e^x \quad y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 0, \quad y'''(0) = 0$$

$$D^4 - D^3 = D^2(D-1)[x + e^x]$$

$$\therefore y_c = C_1 + C_2 x + C_3 x^2 + C_4 e^x \quad \text{since } D^4 - D^3 = D^2(D-1)$$

based on  $D^2(D-1)$ ,  $y_p = C_5 x^3 + C_6 x^4 + C_7 x e^x$

now let's take  $y_p'$ ,  $y_p''$ , and find  $C_5, C_6, C_7$ .

$$y_p' = 3C_5 x^2 + 4C_6 x^3 + C_7 x e^x + C_7 e^x$$

$$y_p'' = 6C_5 x + 12C_6 x^2 + C_7 x e^x + 2C_7 e^x$$

$$y_p''' = 6C_5 + 24C_6 x + C_7 x e^x + 3C_7 e^x; \quad y^{(4)} = 24C_6 + C_7 x e^x + 4C_7 e^x$$

Substituting into  $y^{(4)} - y''' = x + e^x$ :

$$\underbrace{24C_6 + C_7 x e^x + 4C_7 e^x}_{y^{(4)}} - \underbrace{6C_5 - 24C_6 x - C_7 x e^x - 3C_7 e^x}_{y'''} = x + e^x$$

$$\begin{cases} \therefore 24C_6 - 6C_5 = 0 \\ -24C_6 = 1 \\ C_6 = -\frac{1}{24} \end{cases} \quad \begin{cases} 4C_7 e^x - 3C_7 e^x = C_7 e^x \\ \therefore C_7 = 1 \end{cases}$$

$$\therefore 4C_6 - C_5 = 0 \quad \therefore 4\left(-\frac{1}{24}\right) - C_5 = 0$$

$$\boxed{-\frac{1}{6} = C_5}$$

Coef of  $y_p$ :

$$y_p = -\frac{1}{6} x^3 - \frac{1}{24} x^4 + x e^x$$

General solution:

$$y(x) = y_c + y_p = C_1 + C_2 x + C_3 x^2 + C_4 e^x - \frac{1}{6} x^3 - \frac{1}{24} x^4 + x e^x$$

$$\text{Find } y'(x) = C_2 + 2C_3 x + C_4 e^x - \frac{1}{2} x^2 - \frac{1}{6} x^3 + x e^x + e^x$$

$$y''(x) = 2C_3 + C_4 e^x - x - \frac{1}{2} x^2 + x e^x + 2e^x$$

$$y'''(x) = C_4 e^x - 1 - x + x e^x + 3e^x$$

$\Rightarrow$

Initial values:

$$y(0) = 0 \quad \therefore \quad C_1 + C_4 = 0$$

$$y'(0) = 0 \quad \therefore \quad C_2 + C_4 + 1 = 0$$

$$y''(0) = 0 \quad \therefore \quad 2C_3 + C_4 + 2 = 0$$

$$y'''(0) = 0 \quad C_4 - 1 + 3 = 0$$

$$\therefore \quad \underline{C_4 = -2}$$

$$2C_3 + (-2) + 2 = 0 \quad \therefore \quad \underline{C_3 = 0}$$

$$C_2 + C_4 + 1 = 0 \quad \therefore \quad C_2 - 2 + 1 = 0$$

$$\underline{\underline{C_2 = 1}}$$

$$C_1 + C_4 = 0$$

$$C_1 - 2 = 0 \quad \underline{C_1 = 2}$$

replacing constants into  $y(x)$ :

$$y(x) = 2 + x - 2e^x + \frac{1}{6}x^3 - \frac{1}{24}x^4 + xe^x$$

