Two limits

Examples

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1) $\lim_{x \to -\infty} \dfrac{4x^3 + 3x^2}{x - 6x^2}$

Proceed by dividing each term in the numerator and denominator by the highest power of x in the denominator:

$$\lim_{x \to -\infty} \frac{\frac{4x^3}{x^2} + \frac{3x^2}{x^2}}{\frac{x}{x^2} - \frac{6x^2}{x^2}} \quad \Longrightarrow \quad \lim_{x \to -\infty} \frac{4x + 3}{\frac{1}{x} - 6}$$

Now, think of the numerator: the limit of $4x + 3$ as $x \to -\infty$ is $-\infty$. Recall that $-\infty$ is not a real number, it represents the idea that a quantity **keeps decreasing without bound** (for $+\infty$, **quantity that keeps increasing without bound**). In short, in this limit the numerator *tends or approaches* $-\infty$.

What about the denominator?

The $\lim_{x \to -\infty} \dfrac{1}{x} = 0$, and $0 - 6 = -6$; therefore, the quotient of the limits, $\dfrac{-\infty}{-6} = \infty$. Why? dividing *infinity* over 6 is still *infinity*, and, of course, negative divided by negative is positive.

2) $\lim_{x \to \infty} \sqrt{x^2 + 2x} - \sqrt{x^2 - 8x}$

Multiplying by the conjugate:

$$\lim_{x \to \infty} \sqrt{x^2 + 2x} - \sqrt{x^2 - 8x} \cdot \frac{\sqrt{x^2 + 2x} + \sqrt{x^2 - 8x}}{\sqrt{x^2 + 2x} + \sqrt{x^2 - 8x}} = \frac{x^2 + 2x - (x^2 - 8x)}{\sqrt{x^2 + 2x} + \sqrt{x^2 - 8x}} = \frac{10x}{\sqrt{x^2 + 2x} + \sqrt{x^2 - 8x}}$$

Now, dividing top and bottom by the largest power of x in the denominator, $\sqrt{x^2} = |x| = x$ (since $x \to +\infty$):

$$\lim_{x \to \infty} \frac{\frac{10x}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{2x}{x^2}} + \sqrt{\frac{x^2}{x^2} - \frac{8x}{x^2}}} \quad \Longrightarrow \quad \lim_{x \to \infty} \frac{10}{\sqrt{1 + \frac{2}{x}} + \sqrt{1 - \frac{8}{x}}} \quad \text{since} \quad \lim_{x \to \infty} \frac{2}{x} = 0 \quad \text{and} \quad \lim_{x \to \infty} \frac{8}{x} = 0, \ \text{then,}$$

$$\lim_{x \to \infty} \frac{10}{\sqrt{1 + 0} + \sqrt{1 - 0}} = \frac{10}{1 + 1} = 5$$