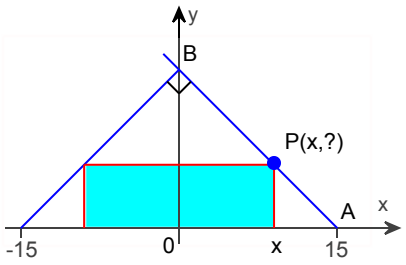
Student: _____
Date: _____

Instructor: Carlos Sotuyo
Course: MAC 2311 – Calculus and Analytical Geometry I

Assignment: Section 4.6 Enhanced Assignment

1. The given figure shows a rectangle inscribed in an isosceles right triangle whose hypotenuse is 30 units long.
(a) Express the y-coordinate of P in terms of x. (Hint: Write an equation for the line AB.)
(b) Express the area of the rectangle in terms of x.
(c) What is the largest area the rectangle can have, and what are its dimensions?



(a) The expression for the y-coordinate of P is **15 − x** .
(Type an expression using x as the variable.)

(b) The expression for the area of the rectangle is **2x(15 − x)** .
(Type an expression using x as the variable.)

(c) The largest area the rectangle can have is $\frac{225}{2}$ square units.
(Type an integer or a simplified fraction.)

What are the dimensions of the rectangle having the largest area?

- ○ **A.** $\frac{15}{2} \times 30$ units
- ○ **B.** 15×30 units
- ◉ **C.** $\mathbf{15 \times \frac{15}{2}}$ **units**

2. A rectangle is constructed with its base on the x-axis and its upper two vertices on the parabola $y = 36 - x^2$. What are the dimensions of the rectangle with the maximum area? What is the area?

The shorter dimension of the rectangle is **$4\sqrt{3}$** and the longer dimension is **24** .
(Type an exact answer, using radicals as needed.)

The area of the rectangle is **$96\sqrt{3}$** .
(Type an exact answer, using radicals as needed.)

3. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 1200 m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions?

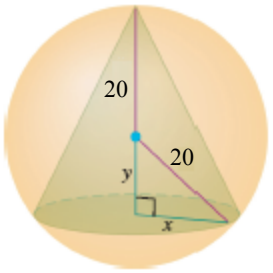
The maximum area of the rectangular plot is **180,000** (1) _____

The length of the shorter side of the rectangular plot is **300** (2) _____

The length of the longer side of the rectangular plot is **600** (3) _____

(1)	(2)	(3)
○ m.	○ m^2.	○ m^2.
○ m^3.	◉ **m.**	◉ **m.**
◉ **m^2.**	○ m^3.	○ m^3.

4. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 20.



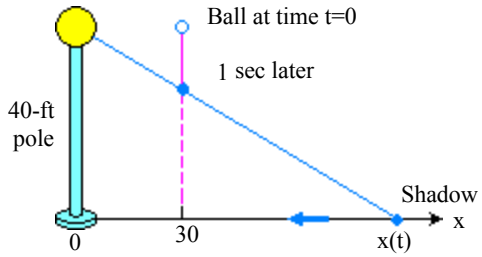
V = $\frac{256{,}000\pi}{81}$ cubic units (Type an exact answer.)

5. What are the dimensions of the lightest open-top right circular cylindrical can that will hold a volume of 216 cm^3?

The radius of the can is $\frac{6}{\sqrt[3]{\pi}}$ cm and its height is $\frac{6}{\sqrt[3]{\pi}}$ cm.

(Type exact answers, using π as needed.)

6. A light shines from the top of a pole 40 ft high. A ball is dropped from the same height from a point 30 ft away from the light. How fast is the shadow of the ball moving along the ground 1 sec later? (Assume the ball falls a distance $s = 16t^2$ in t sec.)



The shadow is moving at a velocity of −150 ft/sec.

(Type an integer or a decimal.)

7. Find the dimensions of a right circular cylinder of maximum volume that can be inscribed in a sphere of radius 35 cm. What is the maximum volume?

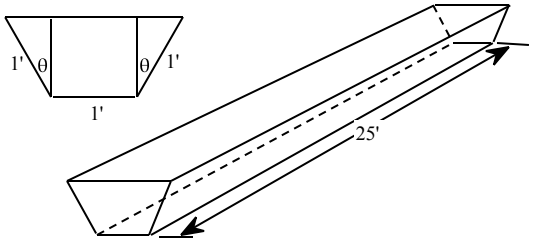
The radius is $\frac{35\sqrt{6}}{3}$ cm, and the height is $\frac{70\sqrt{3}}{3}$ cm.

(Type exact answers, using radicals as needed.)

The volume is $\frac{171{,}500\pi\sqrt{3}}{9}$ cm^3.

(Type an exact answer, using π as needed.)

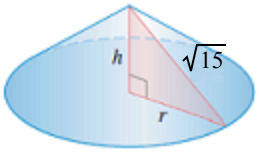
8. The trough in the figure is to be made to the dimensions shown. Only the angle θ can be varied. What value of θ will maximize the trough's volume?



The trough has a maximum volume when the value of θ is $\frac{\pi}{6}$ radians.

(Type an exact answer, using π as needed.)

9. A right triangle whose hypotenuse is $\sqrt{15}$ m long is revolved about one of its legs to generate a right circular cone. Find the radius, height, and volume of the cone of greatest volume that can be made this way.



The height of the cone is $\sqrt{5}$ m.
(Type an exact answer.)

The radius of the cone is $\sqrt{10}$ m.
(Type an exact answer.)

The volume of the cone is $\frac{10\pi\sqrt{5}}{3}$ m^3.
(Type an exact answer, using π as needed.)

10. A wire b units long is cut into two pieces. One piece is bent into an equilateral triangle and the other is bent into a circle. If the sum of the areas enclosed by each part is a minimum, what is the length of each part?

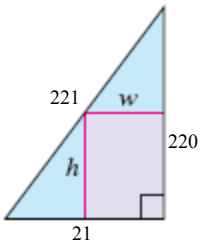
The length of the piece bent into the shape of an equilateral triangle is $\frac{9b}{9+\sqrt{3}\pi}$ units.
(Type an exact answer, using π as needed.)

The length of the piece bent into the shape of a circle is $\frac{b\sqrt{3}\pi}{9+\sqrt{3}\pi}$ units.
(Type an exact answer, using π as needed.)

11. A piece of wire of length 63 is cut into two pieces. One piece is bent into a square and the other is bent into a circle. If the sum of the areas enclosed by each part is a minimum, what is the length of each part?

To minimize the combined area, the wire should be cut so that a length of **27.714** is used for the circle and a length of **35.286** is used for the square.
(Round to the nearest thousandth as needed.)

12. Determine the dimensions of the rectangle of largest area that can be inscribed in the right triangle shown in the accompanying figure.



Let A be the area of the rectangle. Write the objective function in terms of w.

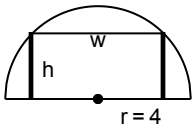
$A(w) = w\left(-\frac{220}{21}w + 220\right)$

The interval of interest of the objective function is **(0,21)**.
(Simplify your answer. Type your answer in interval notation.)

The width of the rectangle with largest area is $\frac{21}{2}$ units.

The height of the rectangle with largest area is **110** units.
(Simplify your answers.)

13. A rectangle is constructed with its base on the diameter of a semicircle with radius 4 and with its two other vertices on the semicircle. What are the dimensions of the rectangle with maximum area?



The rectangle with maximum area has length $\frac{8}{\sqrt{2}}$ units and height $\frac{4}{\sqrt{2}}$ units.

(Type exact answers, using radicals as needed.)

14. **a.** Find the open intervals on which the function is increasing and those on which it is decreasing.
b. Identify the function's local extreme values, if any, saying where they occur.

$h(r) = (r-3)^3$

a. On what open interval(s) is h increasing? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- (●) **A. The function h is increasing on the interval(s) $(-\infty,3),(3,\infty)$.**
  (Type your answer in interval notation. Use a comma to separate answers as needed.)
- ( ) **B.** The function is never increasing.

On what open interval(s) is h decreasing? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ( ) **A.** The function h is decreasing on the interval(s) _____.
  (Type your answer in interval notation. Use a comma to separate answers as needed.)
- (●) **B. The function is never decreasing.**

b. Find each local maximum, if any. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ( ) **A.** The function has (a) local maximum/maxima at the point _____.
  (Type an ordered pair. Use comma to separate answers as needed)
- (●) **B. There is no local maximum.**

Find each local minimum, if any. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ( ) **A.** The function has (a) local minimum/minima at the point _____.
  (Type an ordered pair. Use comma to separate answers as needed)
- (●) **B. There is no local minimum.**

15. What values of a and b make $f(x) = x^3 + ax^2 + bx$ have

a. a local maximum at $x = -7$ and a local minimum at $x = 9$?
b. a local minimum at $x = 12$ and a point of inflection at $x = 1$.

a. For a local maximum at $x = -7$ and a local minimum at $x = 9$, $a =$ **−3** and $b =$ **−189**.

b. For a local minimum at $x = 12$ and a point of inflection at $x = 1$, $a =$ **−3** and $b =$ **−360**.

16. The height (feet) of an object moving vertically is given by $s = -16t^2 + 256t + 192$, where t is in seconds. Find the object's velocity at $t = 4$, its maximum height and when it occurs, and its velocity when $s = 0$.

The velocity of the object at $t = 4$ seconds is **128** ft/second.
(Simplify your answer. Type an integer or a decimal.)

The maximum height occurs at $t =$ **8** seconds.
(Simplify your answer. Type an integer or a decimal.)

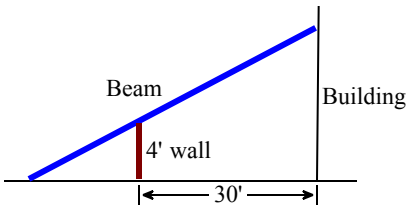
The maximum height is **1216** feet.
(Simplify your answer. Type an integer or a decimal.)

The velocity when $s = 0$ is **−279.04** feet/second.
(Round to the nearest hundredth.)

17. A small island is 2 miles from the nearest point P on the straight shoreline of a large lake. If a woman on the island can row a boat 3 miles per hour and can walk 4 miles per hour, where should the boat be landed in order to arrive at a town 13 miles down the shore from P in the least time?

The boat should be landed $\frac{6}{\sqrt{7}}$ miles down the shore from P.
(Type an exact answer.)

18. The 4-ft wall shown here stands 30 ft from the building. Find the length of the shortest straight beam that will reach to the side of the building from the ground outside the wall.



The length of the shortest beam is **42.5** ft.
(Round to the nearest tenth as needed.)

19. Suppose that $c(x) = 6x^3 - 84x^2 + 17{,}000x$ is the cost of manufacturing x items. Find a production level that will minimize the average cost of making x items.

The production level that minimizes the average cost of making x items is $x =$ **7**.
(Simplify your answer.)

At this level, the average cost of making x items is $ **16,706**.
(Simplify your answer.)

20. A storage shed is to be built in the shape of a box with a square base. It is to have a volume of 810 cubic feet. The concrete for the base costs $3 per square foot, the material for the roof costs $7 per square foot, and the material for the sides costs $4.50 per square foot. Find the dimensions of the most economical shed.

The length of one side of the shed's base is **9** ft.
The height of the shed is **10** ft.