**Student:** ____________________
**Date:** ____________________

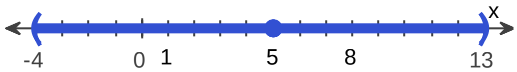
**Instructor:** Carlos Sotuyo
**Course:** MAC 2311 – Calculus and Analytical Geometry I

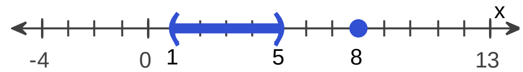
**Assignment:** Section 2.3 Enhanced Assignment

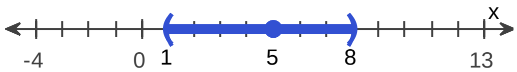
1. Sketch the interval (a,b) on the x-axis with the point c inside the interval. Then find the largest value of $\delta > 0$ such that for all x, $a < x < b$ whenever $0 < |x - c| < \delta$.

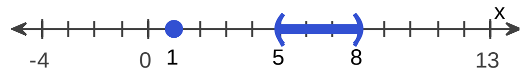
$a = 1, \quad b = 8, \quad c = 5$

Sketch the interval (a,b) on the x-axis with the point c inside the interval. Choose the correct answer below.

○ **A.**



○ **B.**



◉ **C.**



○ **D.**



The largest value of $\delta$ is **3**.
(Type an exact answer in simplified form.)

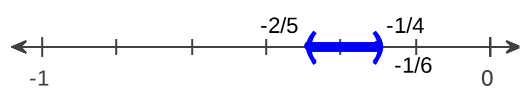
2. Sketch the interval (a,b) on the x-axis with the point c inside. Then find the largest value of $\delta > 0$ such that for all x, $0 < |x - c| < \delta$ implies $a < x < b$.

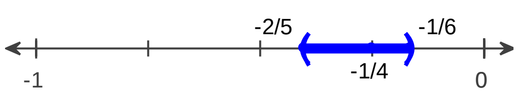
$a = -\frac{2}{5}, \; b = -\frac{1}{6}, \; c = -\frac{1}{4}$

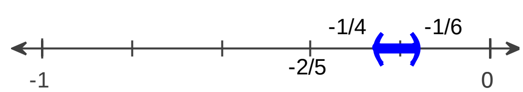
Choose the correct sketch below.

○ **A.**



○ **B.**



◉ **C.**


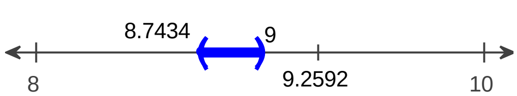

○ **D.**


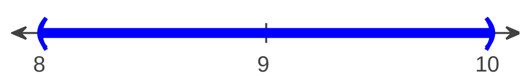

The largest possible value for $\delta$ is $\mathbf{\frac{1}{12}}$.
(Type an integer or a simplified fraction.)

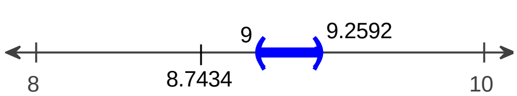
3. Sketch the interval (a,b) on the x-axis with the point c inside. Then find the largest value of $\delta > 0$ such that for all x, $0 < |x - c| < \delta$ implies $a < x < b$.

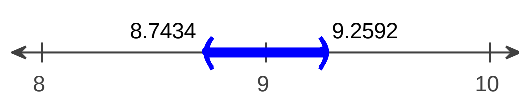
$a = 8.7434, \quad b = 9.2592, \quad c = 9$

Choose the correct sketch below.

○ **A.**


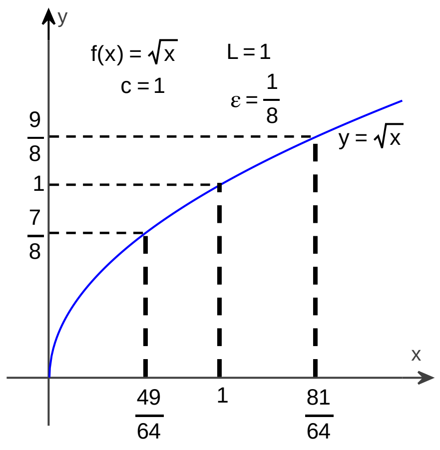

○ **B.**



○ **C.**



◉ **D.**



The largest possible value for $\delta$ is **0.2566**.
(Type an exact answer.)

4. Use the graph below to find the largest value of $\delta > 0$ such that for all x, $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.




The largest value of $\delta$ is $\frac{15}{64}$.

(Simplify your answer. Type an exact answer. Type an integer or a fraction.)

5. For the given function f(x) and values of L, c, and $\varepsilon > 0$ find the largest open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds. Then determine the largest value for $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$, the inequality $|f(x) - L| < \varepsilon$ holds.

$f(x) = 2x + 8, \quad L = 14, \quad c = 3, \quad \varepsilon = 0.1$

The largest open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds is ( **2.95** , **3.05** ).
(Type integers or decimals.)

The largest value of $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$, the inequality $|f(x) - L| < \varepsilon$ holds is **0.05**.
(Simplify your answer. Type an integer or a decimal.)

6. For the given function f(x) and values of L, c, and $\varepsilon > 0$ find the largest open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds. Then determine the largest value for $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$, the inequality $|f(x) - L| < \varepsilon$ holds.

$f(x) = \sqrt{10x + 71}, \quad L = 11, \quad c = 5, \quad \varepsilon = 0.07$

The largest open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds is ( **4.8465** , **5.1545** ).
(Round to four decimal places as needed.)

The largest value of $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$, the inequality $|f(x) - L| < \varepsilon$ holds is is **0.1535**.
(Simplify your answer. Round to four decimal places as needed.)

7. For the given function f(x) and numbers L, c, and $\varepsilon > 0$, find the largest open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds. Then give the largest value of $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$ the inequality $|f(x) - L| < \varepsilon$ holds.

$f(x) = \sqrt{27 - x}$, $L = 2$, $c = 23$, $\varepsilon = 0.1$

The largest open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds is ( **22.59** , **23.39** ).
(Type integers or decimals.)

Find the largest value $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$ the inequality $|f(x) - L| < \varepsilon$ holds.

$\delta =$ **0.39** (Simplify your answer. Type an integer or a decimal.)

8. For the given function f(x) and values of L, c, and $\varepsilon > 0$ find the largest open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds. Then determine the largest value for $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$, the inequality $|f(x) - L| < \varepsilon$ holds.

$$f(x) = \frac{1}{x}, \quad L = \frac{1}{2}, \quad c = 2, \quad \varepsilon = 0.0045$$

The largest open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds is $\left( \mathbf{\frac{2000}{1009}}, \mathbf{\frac{2000}{991}} \right)$.

(Type exact answers.)

The largest value of $\delta > 0$ such that $0 < |x - c| < \delta \rightarrow |f(x) - L| < \varepsilon$ is $\mathbf{\frac{18}{1009}}$.

(Type an exact answer.)

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9. For the given function f(x) and numbers L, c, and $\varepsilon > 0$, find the largest open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds. Then give the largest value of $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$ the inequality $|f(x) - L| < \varepsilon$ holds.

$$f(x) = x^2 - 8, \ L = 8, \ c = 4, \ \varepsilon = 1$$

The largest open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds is $\left( \mathbf{\sqrt{15}}, \mathbf{\sqrt{17}} \right)$.
(Type exact answers, using radicals as needed.)

Find the largest value of $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$ the inequality $|f(x) - L| < \varepsilon$ holds.

$\delta = \mathbf{\sqrt{17} - 4}$
(Simplify your answer. Type an exact answer, using radicals as needed.)

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10. For the given function f(x), the point c, and a positive number $\varepsilon$, find $L = \lim_{x \to c} f(x)$. Then find the largest value of $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$$f(x) = 9 - 5x, \ c = 3, \ \varepsilon = 0.04$$

$L = \mathbf{-6}$ (Simplify your answer.)

What is the largest possible value for $\delta$?

$\delta = \mathbf{0.008}$ (Type an exact answer in simplified form.)

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11. For the given function f(x) and the given values of c and $\varepsilon > 0$, find $L = \lim_{x \to c} f(x)$.
Then determine the largest value for $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$$f(x) = \frac{x^2 - 16}{x - 4}, \quad c = 4, \quad \varepsilon = 0.02$$

The value of L is **8**.
(Simplify your answer.)

The largest value of $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$ is **0.02**.
(Simplify your answer. Round to the nearest hundredth as needed.)

12. For the given function f(x), the point c, and a positive number $\varepsilon$, find $L = \lim_{x \to c} f(x)$. Then find the largest value of $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$f(x) = \sqrt{8 - 8x}$, $c = -7$, $\varepsilon = 0.7$

L = **8** (Simplify your answer.)

What is the largest possible value for $\delta$?

$\delta$ = **1.33875** (Type an exact answer in simplified form.)

13. Give an $\varepsilon$-$\delta$ proof of the limit fact.

$$\lim_{x \to 0} (7x - 2) = -2$$

Let $\varepsilon > 0$ be given. Choose the correct answer below.

- **A.** Choose $\delta = \varepsilon$. Then for all x satisfying $0 < |x - 0| < \delta$, the inequality $|(7x - 2) - (-2)| = |7x| < \delta = \varepsilon$ holds.
- **B.** **Choose $\delta = \frac{\varepsilon}{7}$. Then for all x satisfying $0 < |x - 0| < \delta$, the inequality $|(7x - 2) - (-2)| = |7x| = 7|x| < 7\delta = \varepsilon$ holds.**
- **C.** Choose $\delta = 7\varepsilon$. Then for all x satisfying $0 < |x - 0| < \delta$, the inequality $|(7x - 2) - (-2)| = |7x| = 7|x| < \frac{\delta}{7} = \varepsilon$ holds.
- **D.** Choose $\delta = \frac{\varepsilon}{2}$. Then for all x satisfying $0 < |x - 0| < \delta$, the inequality $|(7x - 2) - 7x| = |-2x| = 2|x| < 2\delta = \varepsilon$ holds.

14. Prove that $\lim_{x \to 7} \sqrt{x-6} = 1$.

For a function f(x) that is defined in an open interval about c, except possibly at c itself, the limit of f(x) as x approaches c is the number L if, for every number $\varepsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all x, $0 < |x - c| < \delta$ implies that $|f(x) - L| < \varepsilon$.

To prove the given limit statement, it is necessary to show that for all x, if $0 < |x - \underline{\mathbf{7}}| < \delta$, then $|\sqrt{x-6} - \underline{\mathbf{1}}| < \varepsilon$.

Solve the inequality $|\sqrt{x-6} - 1| < \varepsilon$ to find an open interval about c = 7 on which this inequality holds for all $x \neq c$.

$$\begin{aligned} & |\sqrt{x-6} - 1| < \varepsilon \\ -\varepsilon < \ & \sqrt{x-6} - 1 < \varepsilon \\ 1 - \varepsilon < \ & \sqrt{x-6} < \underline{\mathbf{1 + \varepsilon}} \\ (1-\varepsilon)^2 < \ & x - 6 < (1+\varepsilon)^2 \\ \underline{\mathbf{7 - 2\varepsilon + \varepsilon^2}} < \ & x < 7 + 2\varepsilon + \varepsilon^2 \end{aligned}$$

The inequality $|\sqrt{x-6} - 1| < \varepsilon$ holds for all $x \neq 7$ in the open interval $(7 - 2\varepsilon + \varepsilon^2, 7 + 2\varepsilon + \varepsilon^2)$. Now, find a value of $\delta > 0$ that places the centered interval $(7 - \delta, 7 + \delta)$ inside the interval $(7 - 2\varepsilon + \varepsilon^2, 7 + 2\varepsilon + \varepsilon^2)$.

Take $\delta$ to be the distance from c = 7 to the nearer endpoint of $(7 - 2\varepsilon + \varepsilon^2, 7 + 2\varepsilon + \varepsilon^2)$.

$$\delta = \min\left\{ \left|7 - \left(7 - 2\varepsilon + \varepsilon^2\right)\right|, \left|7 - \left(7 + 2\varepsilon + \varepsilon^2\right)\right| \right\}$$
$$\delta = \min\left\{ |2\varepsilon - \varepsilon^2|, |-2\varepsilon - \varepsilon^2| \right\}$$
$$\delta = |\underline{\mathbf{2\varepsilon - \varepsilon^2}}|$$

Thus, if $\delta$ has a value of $|2\varepsilon - \varepsilon^2|$ or any smaller positive value, the inequality $0 < |x - 7| < \delta$ automatically places x between $7 - 2\varepsilon + \varepsilon^2$ and $7 + 2\varepsilon + \varepsilon^2$ to make $|\sqrt{x-6} - 1| < \varepsilon$.

Since there exists such a value of $\delta$, the limit as x approaches 7 of the function $\sqrt{x-6}$ is 1.

15.

Prove that $\lim_{x \to 7} f(x) = 49$ if $f(x) = \begin{cases} x^2 & x \neq 7 \\ 6 & x = 7 \end{cases}$.

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For a function f(x) that is defined in an open interval about c, except possibly at c itself, the limit of f(x) as x approaches c is the number L if, for every number $\varepsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all x, $0 < |x - c| < \delta$ implies that $|f(x) - L| < \varepsilon$.

To prove the given limit statement, it is necessary to show that for all x, if $0 < |x - \underline{\mathbf{7}}| < \delta$, then $|\underline{\mathbf{x^2}} - 49| < \varepsilon$.

Solve the inequality $|x^2 - 49| < \varepsilon$ to find an open interval about c = 7 on which this inequality holds for all $x \neq c$.

$$\begin{aligned} & |x^2 - 49| & < \varepsilon \\ -\varepsilon < \; & x^2 - 49 & < \varepsilon \\ 49 - \varepsilon < \; & x^2 & < \underline{\mathbf{49 + \varepsilon}} \\ \underline{\mathbf{\sqrt{49 - \varepsilon}}} < \; & |x| & < \sqrt{49 + \varepsilon} \\ \sqrt{49 - \varepsilon} < \; & x & < \sqrt{49 + \varepsilon} \end{aligned}$$

Assume that $\varepsilon < 49$.

The inequality $|x^2 - 49| < \varepsilon$ holds for all $x \neq 7$ in the open interval $\left(\sqrt{49 - \varepsilon}, \sqrt{49 + \varepsilon}\right)$, where $\varepsilon < 49$. Now, find a value of $\delta > 0$ that places the centered interval $(7 - \delta, 7 + \delta)$ inside the interval $\left(\sqrt{49 - \varepsilon}, \sqrt{49 + \varepsilon}\right)$.

Take $\delta$ to be the distance from c = 7 to the nearer endpoint of $\left(\sqrt{49 - \varepsilon}, \sqrt{49 + \varepsilon}\right)$.

$$\delta = \min\left\{7 - \sqrt{49 - \varepsilon}, \underline{\mathbf{\sqrt{49 + \varepsilon} - 7}}\right\}$$

Thus, for $\varepsilon < 49$, if $\delta$ has a value of $\min\left\{7 - \sqrt{49 - \varepsilon}, \sqrt{49 + \varepsilon} - 7\right\}$ or any smaller positive value, the inequality $0 < |x - 7| < \delta$ automatically places x between $\sqrt{49 - \varepsilon}$ and $\sqrt{49 + \varepsilon}$ to make $|x^2 - 49| < \varepsilon$.

If $\varepsilon \geq 49$, take $\delta$ to be the distance from c = 7 to the nearer endpoint of $\left(0, \sqrt{49 + \varepsilon}\right)$.

$$\delta = \min\left\{\underline{\mathbf{7}}, \sqrt{49 + \varepsilon} - 7\right\}$$

Since there exists a value of $\delta$ such that $0 < |x - 7| < \delta$ that makes the inequality $|x^2 - 49| < \varepsilon$ true for all x, the limit as x approaches 7 of the function $\begin{cases} x^2 & x \neq 7 \\ 6 & x = 7 \end{cases}$ is 49.

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16. Prove that $\lim_{x\to 3} \frac{2}{x} = \frac{2}{3}$.

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For a function f(x) that is defined in an open interval about c, except possibly at c itself, the limit of f(x) as x approaches c is the number L if, for every number $\varepsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all x, $0 < |x - c| < \delta$ implies that $|f(x) - L| < \varepsilon$.

To prove the given limit statement, it is necessary to show that for all x, if $0 < |x - \underline{\mathbf{3}}| < \delta$, then $\left|\frac{2}{x} - \underline{\mathbf{\frac{2}{3}}}\right| < \varepsilon$.

Solve the inequality $\left|\frac{2}{x} - \frac{2}{3}\right| < \varepsilon$ to find an open interval about $c = 3$ on which this inequality holds for all $x \neq c$.

$$\left|\frac{2}{x} - \frac{2}{3}\right| < \varepsilon$$
$$-\varepsilon < \frac{2}{x} - \frac{2}{3} < \varepsilon$$
$$\frac{2}{3} - \varepsilon < \frac{2}{x} < \underline{\mathbf{\frac{2}{3} + \varepsilon}}$$
$$x\left(\frac{2}{3} - \varepsilon\right) < 2 < x\left(\frac{2}{3} + \varepsilon\right)$$
$$\underline{\mathbf{\frac{6}{2 + 3\varepsilon}}} < x < \frac{6}{2 - 3\varepsilon}$$

The inequality $\left|\frac{2}{x} - \frac{2}{3}\right| < \varepsilon$ holds for all $x \neq 3$ in the open interval $\left(\frac{6}{2 + 3\varepsilon}, \frac{6}{2 - 3\varepsilon}\right)$. Now, find a value of $\delta > 0$ that places the centered interval $(3 - \delta, 3 + \delta)$ inside the interval $\left(\frac{6}{2 + 3\varepsilon}, \frac{6}{2 - 3\varepsilon}\right)$.

Take $\delta$ to be the distance from $c = 3$ to the nearer endpoint of $\left(\frac{6}{2 + 3\varepsilon}, \frac{6}{2 - 3\varepsilon}\right)$.

$$\delta = \min\left\{\left|3 - \left(\frac{6}{2 + 3\varepsilon}\right)\right|, \left|3 - \left(\frac{6}{2 - 3\varepsilon}\right)\right|\right\}$$
$$\delta = \min\left\{\left|\frac{9\varepsilon}{2 + 3\varepsilon}\right|, \left|\frac{-9\varepsilon}{2 - 3\varepsilon}\right|\right\}$$
$$\delta = \left|\underline{\mathbf{\frac{9\varepsilon}{2 + 3\varepsilon}}}\right|$$

Thus, if $\delta$ has a value of $\left|\frac{9\varepsilon}{2 + 3\varepsilon}\right|$ or any smaller positive value, the inequality $0 < |x - 3| < \delta$ automatically places x between $\frac{6}{2 + 3\varepsilon}$ and $\frac{6}{2 - 3\varepsilon}$ to make $\left|\frac{2}{x} - \frac{2}{3}\right| < \varepsilon$.

Since there exists such a value of $\delta$, the limit as x approaches 3 of the function $\frac{2}{x}$ is $\frac{2}{3}$.