Practice 23

1) $\int_{-1}^{0} \frac{2t}{(3+t^2)^3}\,dt \implies \int_{-1}^{0} \frac{1}{u^3}\,du = \int_{-1}^{0} u^{-3}\,du = -\frac{1}{2}u^{-2}\Big|_{-1}^{0}$

$u = 3+t^2, \quad du = 2t\,dt$

$= -\frac{1}{2}(3+t^2)^{-2}\Big|_{-1}^{0} = -\frac{1}{2}\frac{1}{(3+t^2)^2}\Big|_{-1}^{0}$

$= -\frac{1}{18} - \left(-\frac{1}{32}\right) = -\frac{7}{288}.$

2) $\int_{1}^{4} \frac{4-\sqrt{x}}{\sqrt{x}}\,dx = -2\int_{1}^{4} u\,du = -2\cdot\frac{u^2}{2}\Big|_{1}^{4} = -(4-\sqrt{x})^2\Big|_{1}^{4}$

$u = 4-\sqrt{x} = 4-x^{1/2}$

$du = -1/2\, x^{-1/2}\,dx$

$-2\,du = \frac{1}{\sqrt{x}}\,dx$

$= -\left[(4-\sqrt{4})^2 - (4-\sqrt{1})^2\right]$

$= -[4-9] = \underline{\underline{5}}$

3) $\int_{0}^{\pi} (1+\cos 9t)^2 \sin 9t\,dt$

$u = 1+\cos 9t \qquad du = -9\sin 9t\,dt$

$-\frac{1}{9}du = \sin 9t\,dt$

$-\frac{1}{9}\int u^2\,du = -\frac{1}{9}\cdot\frac{u^3}{3}\Big|_{0}^{\pi}$

$= -\frac{1}{27}(1+\cos 9t)^3\Big|_{0}^{\pi}$

$= -\frac{1}{27}\left[\underbrace{(1+(-1))^3}_{=0} - \underbrace{(1+1)^3}_{=8}\right] = 8/27$

4) $\int_0^{\pi/2} \frac{\cos x}{(2+4\sin x)^3}\,dx = \frac{1}{4}\int_0^{\pi/2} \frac{1}{u^3}\,du = 1/4 \int u^{-3}\,du = \frac{1}{4}\cdot\frac{u^{-2}}{-2}\Big|_0^{\pi/2}$

$u = 2+4\sin x \qquad du = 4\cos x\,dx, \qquad \frac{1}{4}du = \cos x\,dx$

$= -\frac{1}{8}\left[\frac{1}{(2+4\sin x)^2}\right]\Big|_0^{\pi/2} = -\frac{1}{8}\left[\left(\frac{1}{36}-\frac{1}{4}\right)\right]$

$= -\frac{1}{8}\left(-8/36\right) = 1/36$

5) $\int_0^{\pi/16} \left(1+e^{\tan 4x}\right)\sec^2 4x\,dx$

$u = \tan 4x, \qquad du = 4\sec^2 4x\,dx \Rightarrow \frac{1}{4}du = \sec^2 4x\,dx.$

$= \frac{1}{4}\int_0^{\pi/16}(1+e^u)\,du = \frac{1}{4}(u+e^u)\Big|_0^{\pi/16} = \frac{1}{4}\left[\tan 4x + e^{\tan 4x}\right]\Big|_0^{\pi/16}$

$= \frac{1}{4}\left[(1+e^1)-(0+e^0)\right] = \frac{1}{4}e.$

6) $\int_0^{\pi/20} 20\tan 5x\,dx = 20\int_0^{\pi/20}\frac{\sin 5x}{\cos 5x}\,dx = -\frac{20}{5}\int_0^{\pi/20}\frac{1}{u}\,du$

$u = \cos 5x, \qquad du = -5\sin 5x\,dx$

$-1/5\,du = \sin 5x\,dx$

$= -4\ln|\cos 5x|\Big|_0^{\pi/20} = -4\left[\ln\left|\frac{\sqrt{2}}{2}\right| - \ln(1)\right]$

$= -4\ln\frac{\sqrt{2}}{2} = \ln\left(\frac{\sqrt{2}}{2}\right)^{-4} = \ln\left(\frac{2}{\sqrt{2}}\right)^4 = \ln 4 = \ln 2^2 = \underline{2\ln 2}.$

7) $$\int_0^1 \frac{1}{\sqrt{64-x^2}}\,dx = \int_0^1 \frac{1/\sqrt{64}}{\dfrac{\sqrt{64-x^2}}{\sqrt{64}}}\,dx = \frac{1}{8}\int_0^1 \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx$$

$$= \frac{1}{8}\int_0^1 \frac{1}{\sqrt{1-\left(\frac{x}{8}\right)^2}}\,dx \implies \int_0^1 \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1}u\Big|_0^1$$

$$\boxed{u = x/8 \qquad du = 1/8\,dx}$$

$$= \sin^{-1}(x/8)\Big|_0^1 = \sin^{-1}(1/8) - \underbrace{\sin^{-1}(0)}_{=0}$$

$$= \sin^{-1}(1/8)$$

8) $$\int_{2/7}^{\sqrt{2}/7} \frac{1}{t\sqrt{49t^2-1}}\,dt$$

$$= \int_{2/7}^{\sqrt{2}/7} \frac{1}{t\sqrt{(7t)^2-1}}\,dt = \frac{1}{7}\int_{2/7}^{\sqrt{2}/7} \frac{1}{\frac{u}{7}\sqrt{u^2-1}}\,du$$

$$\boxed{u = 7t, \qquad du = 7\,dt. \quad \frac{1}{7}\,du = dt \quad \therefore \ \frac{u}{7} = t}$$

$$= \frac{7}{7}\left[\sec^{-1}u\right]_{2/7}^{\sqrt{2}/7} = \cos^{-1}\frac{1}{u}\Big|_{2/7}^{\sqrt{2}/7} = \cos^{-1}\left(\frac{1}{7t}\right)\Big|_{2/7}^{\sqrt{2}/7}$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) - \cos^{-1}\left(\frac{1}{2}\right) = -\pi/12$$

$$= \pi/4 - \pi/3 = -\pi/12$$

9) $\int_{-4}^{0} x^3 + x^2 - 6x - 6x \, dx \quad + \quad \int_0^3 6x - (x^3 + x^2 - 6x) \, dx$

$\int_{-4}^{0} x^3 + x^2 - 12x \, dx \quad + \quad \int_0^3 12x - x^3 - x^2 \, dx$

$\frac{x^4}{4} + \frac{x^3}{3} - \frac{12x^2}{2} \Big|_{-4}^{0} \quad + \quad \frac{12x^2}{2} - \frac{x^4}{4} - \frac{x^3}{3} \Big|_0^3$

$0 - \left(64 - \frac{64}{3} - 96\right) \quad + \left(6(3)^2 - \frac{81}{4} - \frac{27}{3}\right) - 0$

$-\left(-\frac{160}{3}\right) + \frac{99}{4} \quad \Rightarrow \quad \frac{160}{3} + \frac{99}{4} = \frac{937}{12}$

10) $\int_1^4 (x-1) - (x^2 - 4x + 3) \, dx = \int_1^4 5x - x^2 - 4 \, dx$

$= \frac{5x^2}{2} - \frac{x^3}{3} - 4x \Big|_1^4 = \left(40 - 64/3 - 16\right) - \left(5/2 - 1/3 - 4\right)$

$= 8/3 - (-11/3) = \underline{\underline{9/2}}$

11) $\int \frac{6}{2+7x} \, dx = \frac{6}{7} \int \frac{1}{u} \, du = 6/7 \ln u + C = \underline{\underline{6/7 \ln|2+7x| + C}}$

$\boxed{u = 2+7x \qquad du = 7dx \qquad 1/7 \, du = dx}$

12) $\int_2^3 \frac{x^4+1}{x^5+5x}\,dx = 1/5 \int_2^3 \frac{1}{u}\,du = 1/5 \ln|x^5+5x|\Big|_2^3$

$u = x^5 + 5x$ $\quad du = 5x^4 + 5\ dx \Rightarrow du = 5(x^4+1)\,dx$

$1/5\,du = (x^4+1)\,dx$

$= \frac{1}{5}\left[\ln(258) - \ln(42)\right] = \frac{1}{5}\ln\left|\frac{258}{42}\right| = 1/5 \ln\left|\frac{43}{7}\right|$

13) $\int \frac{\cos x}{1+6\sin x}\,dx = 1/6 \int \frac{1}{u}\,du = 1/6 \ln|u| + C = 1/6 \ln|1+6\sin x| + C$

$u = 1 + 6\sin x$ $\quad du = 6\cos x\,dx$ $\quad 1/6\,du = \cos x\,dx$