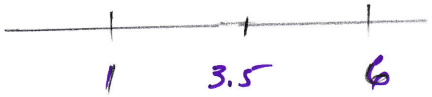
Practice 20

1) $\Delta x = \frac{b-a}{n} = \frac{6-1}{2} = 2.5$



$S = \sum \Delta x f(x_i)$

$S = 2.5 f(3.5) + 2.5(f(6) = 2.5\left[\frac{1}{3.5} + \frac{1}{6}\right]$

$S = 95/84$

2) $\Delta x = \frac{7-4}{4} = 1.$

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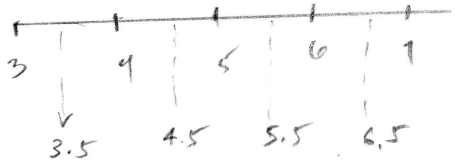

$S = \sum \Delta x f(c_i) \quad \therefore \quad S = 1 f(3.5) + 1 f(4.5) + f(5.5) + f(6.5)$

$S = 3.5^2 + 4.5^2 + 5.5^2 + 6.5^2$

$S = 105$

3) $S = \sum \Delta x f(c_k) \qquad \Delta x = \frac{3-0}{n} = 3/n$

$S = \sum \frac{3}{n}\left[5\left(\frac{3k}{n}\right) + 6\right]$

$S = \sum_{k=1}^{n} \frac{45k}{n^2} + \frac{18}{n}$

$S = \frac{45}{n^2}\sum_{k=1}^{n} k + \frac{18}{n}\sum_{k=1}^{n} 1$

$x_i = a + k\Delta x$

$x_i = 0 + k(3/n)$

$x_i = \frac{3k}{n}$

3)... cont

$$S = \frac{45}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{18}{n}\cdot n$$

$$S = \frac{45}{n^2}\left(\frac{n^2+n}{2}\right) + 18$$

$$S = \frac{45n^2+45n}{2n^2} + 18,$$

$$\therefore \lim_{n\to\infty} S = \frac{45}{2} + 18 = 81/2$$

4) $$S = \sum \Delta x\, f(c_k) = \sum 3/n \left[3\left(\frac{3k}{n}\right)^2 + 4\right]$$

$$S = \sum_{k=1}^{n} \frac{3}{n}\left[\frac{27k^2}{n^2} + 4\right] = \sum_{k=1}^{n}\left(\frac{81k^2}{n^3} + \frac{12}{n}\right)$$

$$S = \frac{81}{n^3}\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 12/n = \frac{81}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + 12$$

$$S = \frac{81}{6n^3}\left(2n^3+3n^2+n\right) + 12$$

$$S = \frac{162n^3 + 243n^2 + 81n}{6n^3} + 12$$

$$\lim_{n\to\infty} S = \frac{162}{6} + 12 = \underline{39}$$