Practice 06

Calculus I

Answers to questions 14 to 22.

14. The velocity function, derivative of position function is $v(t) = 14t + 4$; therefore, $v(t) = 0$ at $t = -\frac{2}{7}$ seconds; that is, during the time interval [0,2] the particle does not change direction; then the displacement is given by position of particle at time two seconds minus position of particle at time 0 seconds: $s(2) - s(0) = 45 - 9 = 36$m.
Average velocity is displacement/time elapsed: $\frac{36}{2} = 18\ m/s$.

15. Speed $=|v(t)|$, and $v(t) = 14t + 2$, at the end of the time interval $t = 2$; therefore $v(2) = 30\ m/s$, –notice that in this case speed = velocity since $v(2) > 0$. Acceleration, derivative of velocity function, $a(t) = 14$. That is, acceleration is constant, does not depend on time.

16. This is a graph of position with respect to time. The particle (body) moves forward whenever the velocity is positive; since velocity is the derivative of the position function, in those interval where the slope is positive, the velocity is positive, the particle is moving forward. That occurs from 0 to 1, from 3 to 4, from 5 to 6 and from 9 to 10.

17. This is a graph of velocity with respect to time. The acceleration is zero when velocity is constant. That occurs from time 2 to 3 and from 5 to 6.

18. A particle changes direction at time t when velocity is equal to zero, and acceleration is different from zero. Of course, when both, velocity and acceleration are equal to zero, the particle stops. In this case $v(t) = 2t–8$, at $t = 4$, $v(4) = 0$, while $a(2) = 2$; that is, at $t = 4$ seconds the particle changes direction.

19. For the given position function $v(t) = 3t^2 - 42t + 144$ and $a(t) = 6t - 42$; factoring velocity function, $v(t) = 3(t-8)(t-6)$; $v(t) = 0$, at $t = 8$ and $t = 6$, find $a(6) = -6$ and $a(8) = 6$.

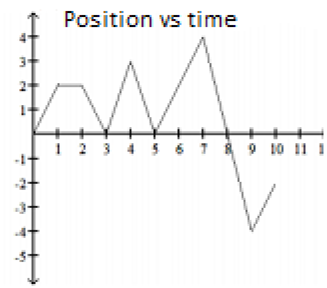
20. In order to find the distance traveled by the particle we need to find when the particle change direction. Find $v(t) = 3t^2 - 30t + 48 = 3(t - 8)(t - 2)$, that is, $v(t) = 0$ at $t = 2$ and $t = 8$. So, at time 2 seconds and 8 seconds the particle changes direction. The question asks for distance traveled from $t = 0$ to $t = 3$; so we need to find distance traveled from 0 to 2, and from 2 to 3.
Calculate position at $t = 0$, $s(0) = 0$; then $s(2) = 44$; at $t = 3$, the position is $s(3) = 36$. Let's put these calculations in context: at $t = 0$, the particle is located at zero; after two seconds, reaches position 44 –the particle moved forward. Remember that at $t = 2$ it changes direction; therefore, it moves backwards, and at $t = 3s$ is located at position 36. Total distance traveled: 44 units forward, and then from 44 to 36, 8 units backwards for a total of 52 total units –units in this problem are meters.

21. The body moves backwards in those time intervals in which velocity is negative. Let's solve the inequality $t^2 - 8t + 7 < 0$, factors: $(t - 7)(t - 1) < 0$; create a sign chart using the number line: we find that from zero to 1, and from 7 to positive infinity $v(t) > 0$, or particle moves forward; from 1 to 7, $v(t) < 0$, that is, it moves backwards.

Big picture: from 0 to 1, the particle moves forward, at $t = 1$ stops and change direction; then from 1 to 7 it moves backwards, at $t = 7$ stops again and changes direction and moves forward.

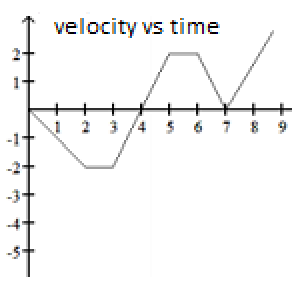
22. Velocity increases when acceleration is positive. For the given $v(t)$, $a(t) = 2t - 6$, and $2t - 6 > 0$ for $t > 3$ secs.

Notes on graphs depicted on question 16 and 17:



16. Description:

This is a graph of position with respect to time. From $t = 1$ to $t = 2$ the particle moves forward, velocity is constant and it is equal to the slope of the line segment. At $t = 2$ the particle reaches position 2 and for a second, it does not move, velocity is zero. From $t = 2$ to $t = 3$ the particle moves backwards, –velocity or slope of the line is negative, and at $t = 3$ returns to position 0. At $t = 3$ it changes direction, it moves again forward (velocity, slope of the position function is positive). At $t = 4$ reaches position 3, and changes direction again reaching position 0 at $t = 5$. Again the particle moves forward from $t = 5$ to 7, reaching position 4 at $t = 7$. At $t = 7$ it changes direction once again and this time around after one second or $t = 8$ crosses position zero and keeps moving backwards reaching position -4 at $t = 9$. Tired of going backwards, it changes direction and for another second, from 9 to 10, moves forward reaching position -2 at $= 10$ and then disappears.



17. Description:

Graph of velocity with respect to time. From $t = 1$ to $t = 4$ the particle moves backwards, since velocities are negatives during this time interval. Specifically, from $t = 1$ to $t = 2$ the velocity decreases, but the speed, absolute value of velocity, increases; that is, the particle speeds up. Notice that during this interval the acceleration is negative. At $t = 2$ velocity is equal to -2, and for a second, from $t = 2$ to $t = 3$, there is no acceleration and both speed and velocity remain constant. From $t = 3$ to $t = 4$, velocity increases, speed decreases, acceleration is positive, the particle slows down reaching velocity and speed of zero at $t = 4$. Since velocity is zero at $t = 4$, but acceleration is not equal to zero (the slope of the line at that point is positive) the particle changes direction, moves forwards, velocity turns positive, speed also increases; that is, again the particle speeds up. From $t = 5$ to $t = 6$ acceleration is zero, velocity is constant and equal to 2. Form $t = 6$ to $t = 7$, velocity and speed decreases, acceleration is negative, the particle slows down reaching velocity equal zero at $t = 7$, that is, the particle stops, and at $t = 7$ the particle speeds up, move forward again, acceleration becomes is positive and at $t = 9$ disappears in space.