Practice 01 MAC2311

Find the average rate of change of the function over the given interval.

1) $y = x^2 + 4x$, $[3, 8]$

$\boxed{a = 3, \quad b = 8. \quad f(3) = 21, \quad f(8) = 96}$

Average rate of change $= \dfrac{f(b) - f(a)}{b - a} = \dfrac{96 - 21}{8 - 3} = \dfrac{75}{5} = 15.$

2) $y = \sqrt{2x}$, $[2, 8]$

ave rate of change $= \dfrac{f(b) - f(a)}{b - a} = \dfrac{4 - 2}{8 - 2} = 2/6 = 1/3$

$\boxed{a = 2, \quad b = 8 \quad f(2) = \sqrt{4} = 2 \quad f(8) = \sqrt{16} = 4}$

3) $h(t) = \sin(5t)$, $\left[0, \dfrac{\pi}{10}\right]$

$a = 0 \quad \sin(0) = 0$

$b = \pi/10 \quad \sin 5(\pi/10) = \sin \pi/2 = 1$

ave rate of change $= \dfrac{h(b) - h(a)}{b - a} = \dfrac{1 - 0}{\pi/10 - 0} = \dfrac{1}{\pi/10} = 10/\pi$

Show that the slope of the tangent line at P(4,36) for the curve described by f(x) is m=13; and, therefore, the equation of the line at that point is $y = 13x - 16$

4) $f(x) = x^2 + 5x$

Slope, $m = \dfrac{\Delta y}{\Delta x}$

$\Delta x = x_2 - x_1$

for $x_2 = x_1 + h$

$\Delta x = (x_1 + h) - x_1 = h.$

$m = \dfrac{\Delta y}{\Delta x} = \dfrac{f(x+h) - f(x)}{h} = \dfrac{(x+h)^2 + 5(x+h) - (x^2 + 5x)}{h}$

$m = \dfrac{x^2 + 2xh + h^2 + 5x + 5h - x^2 - 5x}{h} = \dfrac{2xh + h^2 + 5h}{h} = 2x + h + 5$

as $h \to 0$

$m = 2x + 5 = 2(4) + 5 = 13$

$m = 13.$

eqn: point-slope formula:

$y - y_1 = m(x - x_1)$

$m = 13, \quad x_1 = 4 \quad y_1 = 36$

$y - 36 = 13(x - 4)$

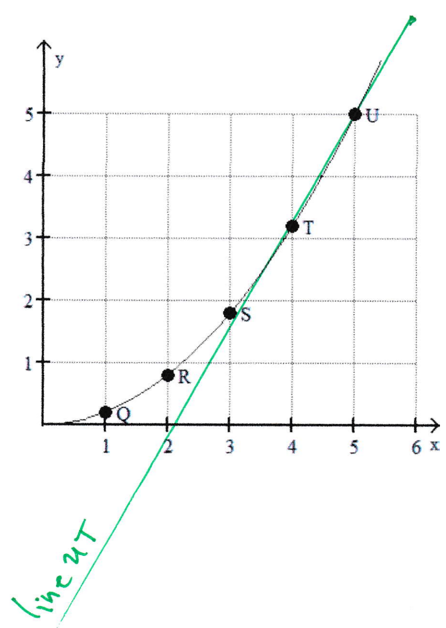
$y - 36 = 13x - 52$

$y = 13x - 52 + 36$

$\boxed{y = 13x - 16}$

Use the slopes of the secant line at UT to estimate the rate of change of y at x = 5.

5)



$T(4, 3)$

$U(5, 5)$

$\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\dfrac{\Delta y}{\Delta x} = \dfrac{5 - 3}{5 - 4}$

$= 2.$