Particle Motion

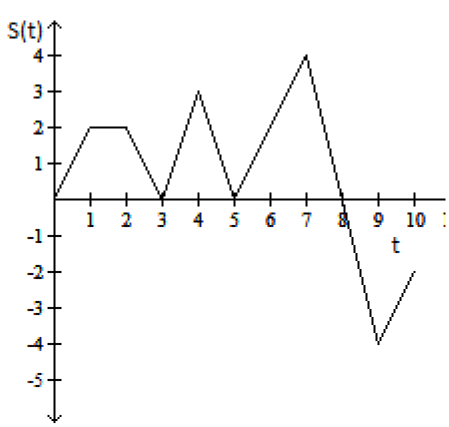
by Carlos Sotuyo

Questions and Answers.

1. The function $s = f(t)$ gives the position of a body moving on a coordinate line, with s in meters and t in seconds.
$s(t) = 7t^2 + 4t + 9; \quad 0 \le t \le 2$ Find the body's displacement and average velocity for the given time interval.
Answer: The displacement of a particle moving in a straight line is the change in its position. If the particle moves from the position S_1 to the position S_2, without changing direction, then its displacement is given by $S_2 – S_1$ over the time interval $[t_1, t_2]$. In this problem the the velocity function, derivative of position function is $v(t) = 14t + 4$; therefore, $v(t) = 0$ at $t = -\frac{2}{7}$ seconds; that is, during the time interval [0,2] the particle does not change direction; then the displacement is given by: $s(2) - s(0) = 45 - 9 = 36$m.
Average velocity is displacement/time elapsed: $\frac{36}{2} = 18 \ m/s$.

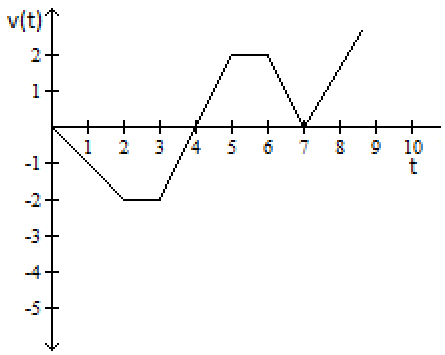
2. For $s(t) = 7t^2 + 2t + 8$; in the interval $0 \le t \le 2$. Find the body's speed and acceleration at the end of the time interval.
Answer: Speed $= |v(t)|$, and $v(t) = 14t + 2$, at the end of the time interval $t = 2$; therefore $v(2) = 30 \ m/s$, –notice that in this case speed = velocity since $v(2) > 0$. Acceleration, derivative of velocity function, $a(t) = 14$. That is, acceleration is constant, does not depend on time.

3. The figure shows the position s of a body moving along a coordinate line as a function of time t. When is the body moving forward?



Answer: This is a graph of position with respect to time. The particle (body) moves forward whenever the velocity is positive; since velocity is the derivative of the position function, in those interval where the slope is positive, the velocity is positive, and, therefore, the particle is moving forward. That occurs from 0 to 1 secs, from 3 to 4 secs, from 5 to 6 secs and from 9 to 10.

4. When is the body's acceleration equal to zero?



Answer: This is a graph of velocity with respect to time. The acceleration is zero when velocity is constant. That occurs from time 2 to 3 secs and from 5 to 6 secs.

5. The position of a body moving on a coordinate line is given by $s(t) = t^2 - 8t + 5$ with s in meters and t in seconds. When, if ever, during the interval $0 \leq t \leq 8$ does the body change direction?

Answer: A particle changes direction at time t when velocity is equal to zero, and acceleration is different from zero. Of course, when both, velocity and acceleration are equal to zero, the particle stops. In this case $v(t) = 2t–8$, at $t = 4$, $v(4) = 0$, while $a(t) = 2$; that is, at $t = 4$ seconds the particle changes direction.

6. At time t, the position of a body moving along the s-axis is $s(t) = t^3 - 21t^2 + 144t$. Find the body's acceleration each time the velocity is zero.

Answer: For the given position function $v(t) = 3t^2 - 42t + 144$ and $a(t) = 6t - 42$; factoring velocity function, $v(t) = 3(t - 8)(t - 6)$; $v(t) = 0$, at $t = 8$ and $t = 6$; therefore,the body's acceleration each time the velocity is zero: $a(6) = -6$ and $a(8) = 6$.

7.At time t, the position of a body moving along the s-axis is $v(t) = t^3 - 15t^2 + 48t$ Find the total distance traveled by the body from $t = 0$ to $t = 3$.

Answer: In order to find the distance traveled by the particle we need to find when the particle change direction. Find $v(t) = 3t^2 - 30t + 48 = 3(t - 8)(t - 2)$, that is, $v(t) = 0$ at $t = 2$ secs and $t = 8$ secs. So, at time 2 seconds and 8 seconds the particle changes direction. The question asks for distance traveled from $t = 0$ to $t = 3$; so we need to find distance traveled from 0 to 2, and from 2 to 3. Calculate position at $t = 0$, $s(0) = 0$; then $s(2) = 44$; at $t = 3$, the position is $s(3) = 36$. Let's put these calculations in context: at $t = 0$, the particle is located at zero; after two seconds, reaches position 44 the particle moved forward. Remember that at $t = 2$ it changes direction; therefore, it moves backwards, and at $t = 3$ secs is located at position 36 m. Total distance traveled: 44 m forward, and then from 44 m to 36 m, 8 m backwards for a total of 52 m.

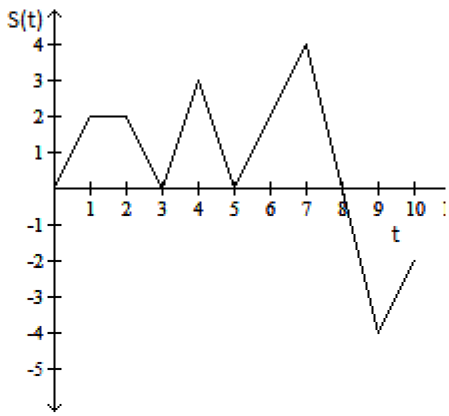
8. At time $t \leq 0$, the velocity of a body moving along the s-axis is $s(t) = t^2 - 8t + 7$. When is the body moving backward?

Answer:

The body moves backwards in those time intervals in which velocity is negative. Let's solve the inequality $t^2 - 8t + 7 < 0$, factors: $(t-7)(t-1) < 0$; create a sign chart using the number line: we find that from zero to 1 sec, and from 7 secs to positive infinity $v(t) > 0$, or particle moves forward; from 1 sec to 7 secs, $v(t) < 0$, that is, it moves backwards.

Big picture: from 0 secs to 1 sec, the particle moves forward, at $t = 1$ stops and change direction; then from 1 sec to 7 secs it moves backwards, at $t = 7$ stops again and changes direction and moves forward.

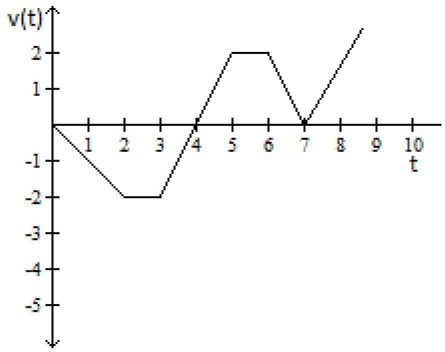
Notes on graphs depicted on question 3 and 4:



3. Description:

This is a graph of position with respect to time. From $t = 1$ to $t = 2$ the particle moves forward, velocity is constant and it is equal to the slope of the line segment. At $t = 2$ the particle reaches position 2 and for a second, it does not move, velocity is zero. From $t = 2$ to $t = 3$ the particle moves backwards, –velocity or slope of the line is negative, and at $t = 3$ returns to position 0. At $t = 3$ it changes direction, it moves again forward (velocity, slope of the position function is positive). At $t = 4$ reaches position 3, and changes direction again reaching position 0 at $t = 5$. Again the particle moves forward from $t = 5$ to 7, reaching position 4 at $t = 7$. At $t = 7$ it changes direction once again and this time around after one second or $t = 8$ crosses position zero and keeps moving backwards reaching position -4 at $t = 9$. Tired of going backwards, it changes direction and for another second, from 9 to 10, moves forward reaching position -2 at $= 10$ and then disappears.

Description of velocity fucntion's graph, next page:



4. Description:

Graph of velocity with respect to time. From $t = 1$ to $t = 4$ the particle moves backwards, since velocities are negatives during this time interval. Specifically, from $t = 1$ to $t = 2$ the velocity decreases, but the speed, absolute value of velocity, increases; that is, the particle speeds up. Notice that during this interval the acceleration is negative. At $t = 2$ velocity is equal to -2, and for a second, from $t = 2$ to $t = 3$, there is no acceleration and both speed and velocity remain constant. From $t = 3$ to $t = 4$, velocity increases, speed decreases, acceleration is positive, the particle slows down reaching velocity and speed of zero at $t = 4$. Since velocity is zero at $t = 4$, but acceleration is not equal to zero (the slope of the line at that point is positive) the particle changes direction, moves forwards, velocity turns positive, speed also increases; that is, again the particle speeds up. From $t = 5$ to $t = 6$ acceleration is zero, velocity is constant and equal to 2. Form $t = 6$ to $t = 7$, velocity and speed decreases, acceleration is negative, the particle slows down reaching velocity equal zero at $t = 7$, that is, the particle stops, and at $t = 7$ the particle speeds up, move forward again, acceleration becomes is positive and at $t = 9$ disappears in space.