Absolute value inequalities

Brief Notes for Calculus

Instructor: Carlos Sotuyo

1. Picture the number line. Now, answer: what is the meaning of $|x| = b$?

The distance of x from zero is b.

That distance is given by the difference between x and *zero*. It may be seen as $|x - 0| = b$.

2. Based on the former definition of $|x|$, what is the meaning of $|x| = 3$?

What are the numbers whose distance form zero is 3. Answer: 3 and -3.

3. In general $|x - c| = b$ answers to *What are the numbers whose distance from c is b?*.

Example: $|x - 5| = 1$ represents the numbers whose distance from 5 is 1.

Of course, the numbers whose distance from 5 is 1 are 4 and 6.

Let's introduce inequalities:

4. How do we read $|x| < 1$?

The interval of real numbers whose distance from zero is less than 1; that is: $-1 < x < 1$

In words, the numbers at a distance of less than 1 from zero are all real numbers in the interval between -1 and 1.

5. Interpret and solve: $|x - 5| < 1$:

Interpretation: *what is the interval of real numbers whose distance from 5 is less than 1?*

Straight forward answer: the interval of real numbers in between 4 and 5 are at a distance from 5 of less than 1. It is written $4 < x < 6$.

Step by step algebraic manipulation:

$$|x - 5| < 1$$

$$-1 < x - 5 < 1$$

$$4 < x < 6.$$

6. We are ready to read the following inequalities:

a. $\left|f(x) - L\right| < \epsilon$

b. $|x - a| < \delta$

Answer:

a) What are function values whose distance from L are less than ϵ *(epsilon)*?

b) What are the numbers x whose distance from a is less than δ *(delta)*?

7. Solve $|x - a| < \delta$ for $a = 3$ and $\delta = 0.01$.

Substituting a and δ:

$|x - 3| < 0.01$

All real numbers at a distance of 0.01 from $3:$ $\Longrightarrow$ $2.99 < x < 3.01$.

Solving inequalities:

1. We all know that if $a < b$ then, $-a > -b$.

Therefore, a compound inequality like,

$1 < -x < 3$ is equivalent to:

$-3 < x < -1$.

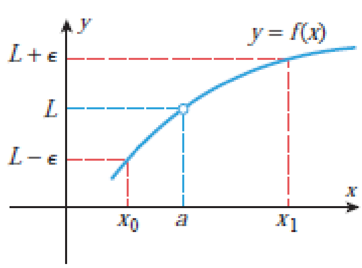
2. We should also know that, if $a < b$ for a and b not equal to zero, then $\dfrac{1}{a} > \dfrac{1}{b}$

So, the following compound inequality:

$2 < \dfrac{1}{x} < 3$ becomes,

$\dfrac{1}{3} < x < \dfrac{1}{2}$

Now we are ready to work on the *epsilon* definition of limit.



See Notes 03.