The Derivative

NOTES

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2.1, 2.2, 2.3: Tangent lines and rates of change. The derivative Function. Introduction to Techniques of Differentiation:

Suppose that x_0 is in the domain of the function f . The tangent line to the curve $y = f(x)$ at the point $P(x_0, f(x_0))$ is the line with equation $y - f(x_0) = m_{tan}(x - x_0)$; where,

$$m_{tan} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

Provided the limit exists. Let $x - x_0 = \Delta x = h$; which implies that $h = x - x_0$; therefore, we rewrite:

$$m_{tan} = \lim_{h \to x_0} \frac{f(x_0 + h) - f(x_0)}{h}$$

Examples:

1. Find the equation of the tangent line to $f(x) = \frac{1}{x}$ at the point $(2, 3)$.
2. Find the equation of the tangent line to $f(x) = x^2$ at the point $(1, 1)$.
3. Find the equation of the tangent line to $f(x) = \sqrt{x}$ at the point $(4, 2)$.

Velocity as change of position with respect to time. Average rate of change. Instantaneous velocity. (Use a graph to distinguish between the two).

The Derivative Function:

Derivative as instantaneous rate of change or *direction of the curve* at a given point:

$$f'(x) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

The *derivative formula of the function with respect to x* may also be written as follows:

$$f'(x) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Use the previous results to find derivatives of, say, $f(x) = x^2$; $f(x) = \frac{1}{x}$ $f(x) = x^3$, etc.

Derive the formula for the derivative of $f(x) = x^n$.

Derivative of a constant function is zero. Why?

Derivative of a constant times a function is the product of the constant times the derivative of a function; derivative of a sum of functions, is equal to the sum of the derivatives. Relates these properties to the properties of limits. BUT, derivative of a product is not equal to the product of the derivatives; neither the quotient of functions is equal to the quotient of the derivatives. These two rules will be studied later in this chapter.

Higher Order derivatives. Notation.