Limits and Continuity

NOTES

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1.1 Limits:

Concept of limit: geometric view. Rational and irrational numbers as limits. Tangent line: find the equation of a tangent line at a point: example, $y = x^2$, at $(1, 1)$.

- Definition: We write $\lim_{x \to a} f(x) = L$ which is read *the limit of f(x) as x approaches a is L*:

$$\lim_{x \to a} f(x) = L$$

Use numerical evidence to conjecture about the value of:

$$\lim_{x \to 1} \frac{x - 1}{\sqrt{x} - 1}$$

Three ways of finding limits: numerically, graphically, analytically.

Use a graph of three functions: a continuous one, and three others with hole discontinuity, jump, and infinite discontinuity to explain how to find limits graphically.

One sided limits: Use example of $f(x) = \dfrac{|x|}{x}$ as x approaches zero from left and right.

Explain infinite limits using vertical asymptotes, example: $f(x) = \dfrac{1}{x - 2}$ and $f(x) = \dfrac{1}{(x - 2)^2}$

$$\lim_{x \to a} f(x) = \infty$$

1.2 Computing limits:

Explain Theorem 1.2.1:

Constant function: $\lim_{x \to a} k = k$; Polynomial: $\lim_{x \to a} x = a$; $\lim_{x \to 0^-} \dfrac{1}{x} = -\infty$; $\lim_{x \to 0^+} \dfrac{1}{x} = +\infty$

Explain Theorem 1.2.2:

(a) The limit of a sum is the sum of the limits.

(b) The limit of a difference is the difference of the limits.

(c) The limit of a product is the product of the limits.

(d) The limit of a quotient is the quotient of the limits, provided the limit of the denominator is not zero.

(e) The limit of an nth root is the nth root of the limit.

Explain using examples:

- Limits of rational functions as x approaches a given value.
- Limits of piece-wise defined functions

1.3 Limits at infinity. End behavior of a function:

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty} x^3 + 2x + 5 = \infty$$

$$\lim_{x\to-\infty} x^3 + 2x + 5 = -\infty$$

In general, explain functions where (as x approaches $-\infty$ and or x approaches $+\infty$):

$$\lim_{x\to\infty} f(x) = \infty$$

$$\lim_{x\to\infty} f(x) = L$$

relates the results to the (odd or even) degree of the polynomial: end behavior.

Work on practice 1 and textbook questions.