Antiderivatives

NOTES 19

Instructor: Carlos Sotuyo

4.8 Antiderivatives:

We have studied how to find the derivative of a function and how to use it to solve a wide range of problems. However, many other problems require that we recover a function from its known derivative (from its known rate of change). For instance, the laws of physics tell us the acceleration of an object falling from an initial height, and we can use this to compute its velocity and its height at any time. More generally, starting with a function f, we want to find a function F whose derivative is f. If such a function F exists, it is called an antiderivative of f.

Definition:

A function F is an antiderivative of f on an interval I if $F(x) = f(x)$ for all x in I.

Example 1: Find an antiderivative for each of the following functions.

a) $y = 3x^2$ b) $y = sec^2 x$

Answers:

a) $y = x^3$ b) $y = \tan x$

Theorem:

If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$F(x) + C$ where C is an arbitrary constant.

Table of Antiderivative formulas, k a nonzero constant:

Functions	Antiderivatives
x^n	$\frac{1}{n+1}x^{n+1} + C, \ n \neq -1$
$\sin kx$	$-\frac{1}{k}\cos kx + C$
$\cos kx$	$\frac{1}{k}\sin kx + C$
$\sec^2 kx$	$\frac{1}{k}\tan kx + C$
$\csc^2 kx$	$-\frac{1}{k}\cot kx + C$
$\sec kx \tan kx$	$\frac{1}{k}\sec kx + C$
$\csc kx \cot kx$	$-\frac{1}{k}\csc kx + C$
e^{kx}	$\frac{1}{k}e^{kx} + C$

Example 2: Find the general antiderivative of each of the following functions.

a) $f(x) = x^5$ b) $f(x) = \sin 2x$ c) $f(x) = e^{3x}$

Table of Antiderivative linearity rules:

Rule	Function	General Antiderivatives
Constant multiple	$kf(x)$	$k(Fx) + C$
Negative rule	$-f(x)$	$-F(x) + C$
Sum of difference	$f(x) + g(x)$	$F(x) + G(x) + C$

Indefinite Integrals:

A special symbol is used to denote the collection of all antiderivatives of a function f.The collection of all antiderivatives of f f is called the indefinite integral of f with respect to x, and is denoted by

$$\int f(x)\ dx$$

The symbol $\int$ is an integral sign. The function f is the integrand of the integral, and x is the variable of integration.

Example: $\int 2x\ dx = x^2 + C$