Monotonic Functions and the First Derivative Test.

NOTES 16

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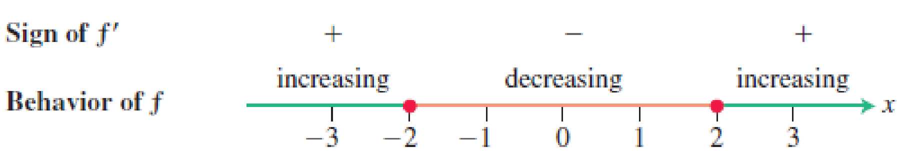
4.3: Monotonic Functions and the First Derivative Test:

We can show that functions with positive derivatives are increasing functions and functions with negative derivatives are decreasing functions. A function that is increasing or decreasing on an interval is said to be monotonic on the interval.
Suppose that f is continuous on $[a, b]$ and differentiable on (a, b).
If $f'(x) > 0$ at each point $x \in (a, b)$, then f is increasing on $[a, b]$.
If $f'(x) < 0$ at each point $x \in (a, b)$, then f is decreasing on $[a, b]$.

Example 1: Find the critical points of $f(x) = x^3 - 12x - 5$ and identify the open intervals on which f is increasing and on which f is decreasing.
The function f is everywhere continuous and differentiable. The first derivative $f'(x) = 3x^2 - 12 = 3(x + 2)(x - 2)$ is zero at $x = -2$ and $x = 2$.
These critical points subdivide the domain of f to create non-overlapping open intervals $(-\infty, -2)$, $(-2, 2)$, and $(2, \infty)$ on which f' is either positive or negative. We determine the sign of f' by evaluating f' at a convenient point in each subinterval.



Interpretation: From $(-\infty, -2)$, the first derivative is positive, (the slopes of the tangent lines to the curve are positive) and therefore the function increases in the interval. At $x = -2$ the first derivative es zero, that is, the tangent line at that point is horizontal (slope, $m = 0$); then in the interval $(-2, 2)$, the derivative is negative, which implies that the slopes of the tangent lines to the curve are negative, so the function is decreasing. Notice that for $x < -2$ the function is increasing, and for $x > -2$ is decreasing; therefore, at $x = -2$ the function has a (local) maximum. For $x > 2$ the function increases again, that is, at $x = 2$ the function reaches a (local) minimum. See the function graph at the end of these notes.

First Derivative Test for Local Extrema:

Suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself.

Moving across this interval from left to right,

1. if f' changes from negative to positive at c then f has a local minimum at c;
2. if f' changes from positive to negative at c, then f has a local maximum at c;
3. if f' does not change sign at c (that is, f' is positive on both sides of c or negative on both sides), then f has no local extremum at c.

Examine the graph of $f(x) = x^3 - 12x - 5$ and identify $(-2, 11)$ as a local maximum and $(2, -21)$ as a local minimum.