Differentiation Rules. Derivative as a Rate of Change.

NOTES 07

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Powers, Multiples, Sums, and Differences:

A simple rule of differentiation is that the derivative of every constant function is zero.

Definition:

If f has the constant value $f(x) = c$, then:

$$\frac{df}{dx} = \frac{d}{dx}(c) = 0$$

Why? Since, the slope of the tangent line to the curve $y = f(x)$ at $x = x_0$ is given by the derivative of $f(x)$, $m = f'(x_0)$, a constant function $y = c$ has slope equal zero for all x.
Analytically, using the definition of the derivative: $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \implies f'(x) = \lim_{h\to 0} \frac{c-c}{h} = 0$

Power Rule:

Definition:

Power Rule: If n is any real number, then:

$$\frac{d}{dx}x^n = nx^{n-1}$$

for all x where the powers x^n and x^{n-1} are defined.

Use the alternative formula for the definition of the derivative, $f'(c) = \lim_{c\to x} \frac{f(c) - f(x)}{c - x}$ and the expansion of $c^n - x^n$ to prove that $\frac{d}{dx}x^n = nx^{n-1}$ is true for all positive integers n.

Derivative Constant Multiple Rule:

Definition:

If u is a differentiable function of x, and c is a constant, then:

$$\frac{d}{dx}(cu) = c\frac{du}{dx}$$

Derivative Sum Rule:

Definition:

If u and v are differentiable functions of x, then their sum $u+v$ is differentiable at every point where u and v are both differentiable. At such points,

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

Derivative Product Rule:

Definition:

If u and v are differentiable at x, then so is their product uv, and

$$\frac{d}{dx}(u \cdot v) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

Derivative Quotient Rule:

Definition:

If u and v are differentiable at x and if $v(x) \neq 0$, then the quotient $\frac{u}{v}$ is differentiable at x, and

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

for simplicity, we write:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$$

Derivative of the Natural Exponential Function:

Definition:

$$\frac{d}{dx}e^x = e^x$$

Second and Higher-Order Derivatives:

If $y = f(x)$ is a differentiable function, then its derivative $f'(x)$ is also a function. If $f'(x)$ is also differentiable, then we can differentiate $f'(x)$ to get a new function of x denoted $f''(x)$. The function $f''(x)$ is called the second derivative of f because it is the derivative of the first derivative. Notation for the second derivative:

$f''(x) = y'' = \dfrac{d^2y}{dx^2} = D^2(f)(x)$

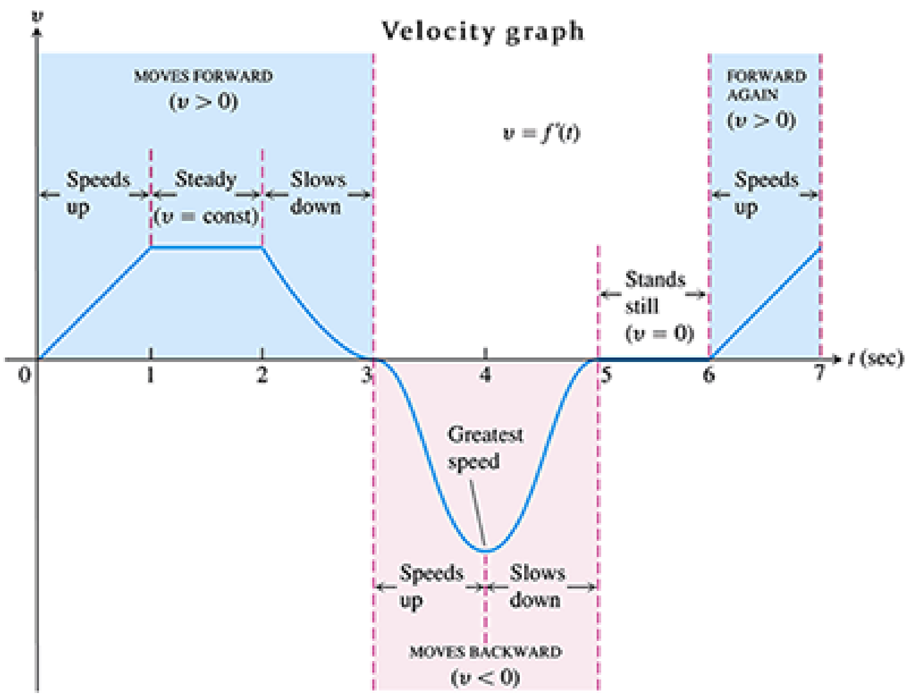
3.4 The Derivative as a Rate of Change:

We can interpret the limit as $h \to 0$ as the rate at which f is changing at the point x.
The instantaneous rate of change of f with respect to x at x_0 is the derivative: $f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$

Motion Along a Line: Displacement, Velocity, Speed, Acceleration, and Jerk:

Definition: Velocity (*instantaneous velocity*) is the derivative of position with respect to time. If a body's position at time t is $s = f(t)$, then the body's velocity at time t is $v(t) = \frac{ds}{dt}$

Speed is the absolute value of velocity: Speed $= |v(t)| = \left|\frac{ds}{dt}\right|$



Notice that:

* Particle moves forward when velocity is positive;

* Particle moves backward when velocity is negative;

* Particle speeds up when both velocity and acceleration have the same sign;

* Particle slows down when velocity and acceleration have opposite signs;

* At a constant velocity there is no acceleration;

* Particle changes direction if velocity is zero, but acceleration is not zero;

* If both, velocity and acceleration are zero, particle stands still (stops).

* Greatest speed: particle speeds up and then slows down.

Acceleration is the derivative of velocity with respect to time. If a body's position at time t is $s = f(t)$, then the body's acceleration at time t is: $a(t) = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$

Jerk is the derivative of acceleration with respect to time: $j = \frac{da}{dt}$