Student: ____________________ **Date:** ____________________	**Instructor:** Carlos Sotuyo **Course:** MAC 2311 – Calculus and Analytical Geometry I	**Assignment:** Section 3.10 Enhanced Assignment

1. Suppose that a dimension x and the area $A = 8x^2$ of a shape are differentiable functions of t. Write an equation that relates $\frac{dA}{dt}$ to $\frac{dx}{dt}$.

$\frac{dA}{dt} = \left(\underline{\quad \mathbf{16x} \quad}\right)\frac{dx}{dt}$

---

2. Assume that $y = 7x$ and $\frac{dx}{dt} = 9$. Find $\frac{dy}{dt}$.

$\frac{dy}{dt} =$ **63** (Simplify your answer.)

---

3. Assume that $x = x(t)$ and $y = y(t)$. Let $y = x^3 + 5$ and $\frac{dx}{dt} = 3$ when $x = 5$.

Find $\frac{dy}{dt}$ when $x = 5$.

$\frac{dy}{dt} =$ **225** (Simplify your answer.)

---

4. Assume that all variables are implicit functions of time t. Find the indicated rate.

$x^2 + 2y^2 + 2y = 9$; $\frac{dx}{dt} = 1$ when $x = 3$ and $y = -1$; find $\frac{dy}{dt}$

$\frac{dy}{dt} =$ **3** (Simplify your answer.)

---

5. The original 24 m edge length x of a cube decreases at the rate of 2 m/min.

a. When x = 2 m, at what rate does the cube's surface area change?
b. When x = 2 m, at what rate does the cube's volume change?

a. When x = 2 m, the surface area is changing at a rate of **−48** m^2/min.
(Type an integer or a decimal. Do not round.)

b. When x = 2 m, the volume is changing at a rate of **−24** m^3/min.
(Type an integer or a decimal. Do not round.)

6. The dimensions x and y of an object are related to its volume V by the formula $V = 9x^2y$.

a. How is $\frac{dV}{dt}$ related to $\frac{dy}{dt}$ if x is constant?

b. How is $\frac{dV}{dt}$ related to $\frac{dx}{dt}$ if y is constant?

c. How is $\frac{dV}{dt}$ related to $\frac{dx}{dt}$ and $\frac{dy}{dt}$ if neither x nor y is constant?

---

a. Complete the equation for when x is constant.

$$\frac{dV}{dt} = \left(\underline{\mathbf{9x^2}}\right)\frac{dy}{dt}$$

b. Complete the equation for when y is constant.

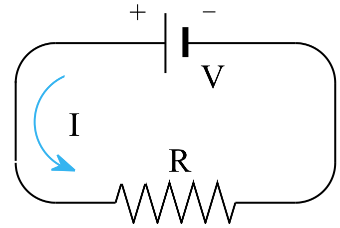
$$\frac{dV}{dt} = \left(\underline{\mathbf{18xy}}\right)\frac{dx}{dt}$$

c. Complete the equation for when neither x nor y is constant.

$$\frac{dV}{dt} = \left(\underline{\mathbf{18xy}}\right)\frac{dx}{dt} + \left(\underline{\mathbf{9x^2}}\right)\frac{dy}{dt}$$

---

7. The voltage V (volts), current I (amperes), and resistance R (ohms) of an electric circuit like the one shown here are related by the equation $V = IR$. Suppose that V is increasing at the rate of 3 volt/sec while I is decreasing at the rate of $\frac{1}{2}$ amp/sec. Let t denote time in seconds. Answer the following questions.



---

a. What is the value of $\frac{dV}{dt}$?

$\frac{dV}{dt} =$ **3** volt/sec (Simplify your answer.)

b. What is the value of $\frac{dI}{dt}$?

$\frac{dI}{dt} =$ $\mathbf{-\frac{1}{2}}$ amp/sec (Simplify your answer.)

c. What equation relates $\frac{dR}{dt}$ to $\frac{dV}{dt}$ and $\frac{dI}{dt}$?

○ **A.** $\frac{dR}{dt} = I\frac{dI}{dt} + V\frac{dV}{dT}$

◉ **B.** $\frac{dI}{d}$

○ **C.** $\frac{dR}{dt} = \frac{1}{I}\frac{dV}{dt} + V\frac{dI}{dt}$

○ **D.** $\frac{dR}{d}$

d. Find the rate at which R is changing when $V = 8$ volts and $I = 4$ amp. Is R increasing or decreasing?

R is changing at **1** ohm/sec. (Simplify your answer.)

Is R increasing or decreasing?

◉ **Increasing**

○ Decreasing

8. Let x and y be differentiable functions of t, and let $s = \sqrt{6x^2 + 4y^2}$ be a function of x and y. Answer the following questions.

a. How is $\frac{ds}{dt}$ related to $\frac{dx}{dt}$ if y is constant?

$$\frac{ds}{dt} = \left(\frac{6x}{\sqrt{6x^2 + 4y^2}} \right) \frac{dx}{dt}$$

b. How is $\frac{ds}{dt}$ related to $\frac{dx}{dt}$ and $\frac{dy}{dt}$ if neither x nor y is constant?

$$\frac{ds}{dt} = \left(\frac{6x}{\sqrt{6x^2 + 4y^2}} \right) \frac{dx}{dt} + \left(\frac{4y}{\sqrt{6x^2 + 4y^2}} \right) \frac{dy}{dt}$$

c. How is $\frac{dx}{dt}$ related to $\frac{dy}{dt}$ if s is constant?

$$\frac{dx}{dt} = \left(-\frac{2y}{3x} \right) \frac{dy}{dt}$$

---

9. When a circular plate of metal is heated in an oven, its radius increases at a rate of 0.03 cm / min. At what rate is the plate's area increasing when the radius is 56 cm?

Write an equation relating the area of the circular plate, A, and the radius, r.

$A = \pi r^2$

Differentiate both sides of the equation with respect to t.

$\frac{dA}{dt} = \left(2\pi r \right) \frac{dr}{dt}$ (Type an expression using r as the variable.)

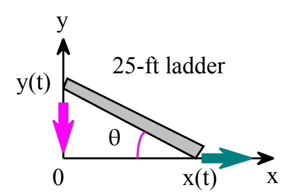
The rate of change of the area is **3.36π** (1) ______

(Type an exact answer in terms of π.)

(1) ○ cm / min.
○ cm.
◉ **cm^2 / min.**

---

10. A 25-ft ladder is leaning against a house when its base starts to slide away. By the time the base is 7 ft from the house, the base is moving away at the rate of 24 ft/sec.
a. What is the rate of change of the height of the top of the ladder?
b. At what rate is the area of the triangle formed by the ladder, wall, and ground changing then?
c. At what rate is the angle between the ladder and the ground changing then?



a. The rate of change of the height of the top of the ladder is **−7** ft/sec.
(Simplify your answer.)

b. The area is changing at **263.5** ft^2/sec.
(Simplify your answer.)

c. The angle is changing at **−1** rad/sec.
(Simplify your answer.)

11. A child flies a kite at a height of 80 ft, the wind carrying the kite horizontally away from the child at a rate of 34 ft / sec. How fast must the child let out the string when the kite is 170 ft away from the child?

The child must let out the string at a rate of **30** ft / sec when the kite is 170 ft away from the child.
(Simplify your answer.)

---

12. Sand falls from a conveyor belt at a rate of 10 m^3 / min onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. How fast are the height and the radius changing when the pile is 5 m high? Answer in centimeters per minute.

The height is changing at a rate of $\frac{45}{2\pi}$ cm / min when the height is 5 m.
(Type an exact answer, using π as needed.)

The radius is changing at a rate of $\frac{30}{\pi}$ cm / min when the height is 5 m.
(Type an exact answer, using π as needed.)

---

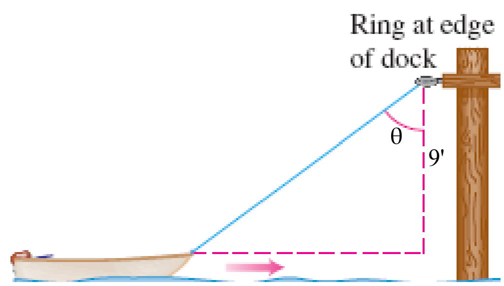
13. A spherical balloon is inflating with helium at a rate of 300π ft^3 / min. How fast is the balloon's radius increasing at the instant the radius is 5 ft? How fast is the surface area increasing?

The balloon's radius is increasing at a rate of **3** ft / min at the instant the radius is 5 ft.
(Simplify your answer.)

The surface area is increasing at a rate of **120π** ft^2 / min at the instant the radius is 5 ft.
(Type an exact answer, using π as needed.)

---

14. A dinghy is pulled toward a dock by a rope from the bow through a ring on the dock 9 ft above the bow. The rope is hauled in at the rate of 1 ft/sec. Complete parts (a) and (b).



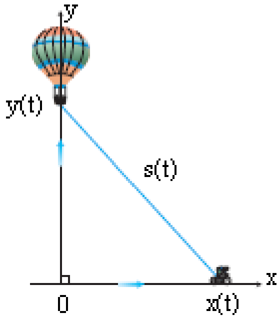
a. How fast is the boat approaching the dock when 15 ft of rope are out?

$-\frac{5}{4}$ ft/sec
(Type an integer or a simplified fraction.)

b. At what rate is the angle θ changing at this instant?

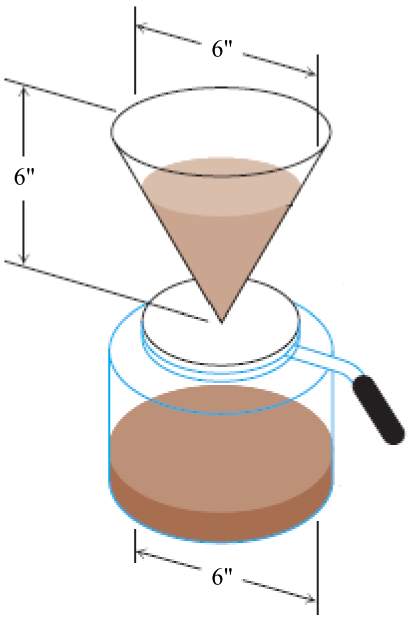
$-\frac{1}{20}$ rad/sec
(Type an integer or a simplified fraction.)

15. A balloon is rising vertically above a level, straight road at a constant rate of 5 ft / sec. Just when the balloon is 25 ft above the ground, a bicycle moving at a constant rate of 10 ft / sec passes under it. How fast is the distance s(t) between the bicycle and balloon increasing 3 seconds later?



s(t) is increasing by **10** ft / sec.
(Simplify your answer.)

16. Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of 6 in.^3 / min. Complete parts (a) and (b).



a. How fast is the level in the pot rising when the coffee in the cone is 4 in. deep?

$\mathbf{\frac{2}{3\pi}}$ (1) ________
(Type an exact answer, using π as needed.)

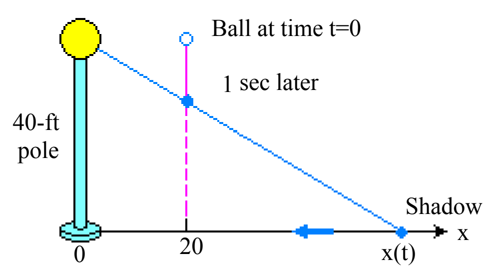
b. How fast is the level in the cone falling then?

$\mathbf{\frac{3}{2\pi}}$ (2) ________
(Type an exact answer, using π as needed.)

(1) ◉ **in. / min**
○ in. • min
○ min / in.
○ in.

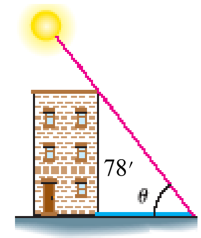
(2) ◉ **in. / min**
○ in. • min
○ min / in.
○ in.

17. A light shines from the top of a pole 40 ft high. A ball is dropped from the same height from a point 20 ft away from the light. How fast is the shadow of the ball moving along the ground 1 sec later? (Assume the ball falls a distance $s = 16t^2$ in t sec.)



The shadow is moving at a velocity of **− 100** ft/sec.
(Type an integer or a decimal.)

18. On a morning of a day when the sun will pass directly overhead, the shadow of a 78-ft building on level ground is 104 ft long. At the moment in question, the angle θ the sun makes with the ground is increasing at the rate of $0.26°$/min. At what rate is the shadow decreasing? Remember to use radians in your calculations. Express your answer in inches per minute.



The shadow is decreasing at **11.8** inches per minute.
(Round to one decimal place as needed.)

---

19. A spherical iron ball 10 in. in diameter is coated with a layer of ice of uniform thickness. If the ice melts at a rate of 11 in.^3 / min, how fast is the thickness of the ice decreasing when it is 4 in. thick? How fast is the outer surface area of ice decreasing?

Write an equation relating the volume of the sphere, V, to the thickness of the ice, x.

$$V = \frac{4}{3}\pi(5+x)^3$$

Differentiate both sides of the equation with respect to t, and solve for $\frac{dx}{dt}$.

$$\frac{dx}{dt} = \left(\frac{1}{4\pi(5+x)^2}\right)\frac{dV}{dt}$$ (Type an expression using x as the variable.)

The thickness of the ice is decreasing at a rate of $\frac{11}{324\pi}$ (1) ______ when the ice is 4 in. thick.

(Type an exact answer, using π as needed.)

Write an equation relating the outer surface area of the ice, A, to the thickness of the ice, x.

$$A = 4\pi(5+x)^2$$

Differentiate both sides of the equation with respect to t.

$$\frac{dA}{dt} = \left(8\pi(5+x)\right)\frac{dx}{dt}$$ (Type an expression using x as the variable.)

The outer surface area of ice is decreasing at a rate of $\frac{22}{9}$ (2) ______ when the ice is 4 in. thick.

(Type an integer or a simplified fraction.)

(1)
- ○ min / in.
- ○ in.
- ◉ **in. / min**

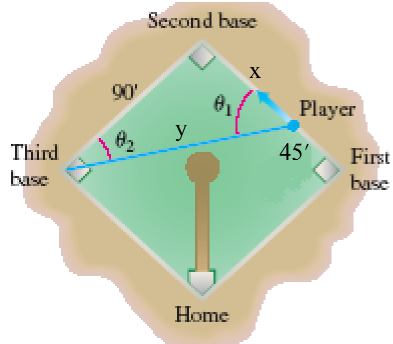
(2)
- ◉ **in.^2 / min**
- ○ in.^2
- ○ min^2 / in.

20. A baseball diamond is a square 90 ft on a side. A player runs from first base to second at a rate of 14 ft/sec.

a. At what rate is the player's distance from third base changing when the player is 45 ft from first base?

b. At what rates are angles θ_1 and θ_2 (see the figure) changing at that time?

c. The player slides into second base at the rate of 15 ft / sec. At what rates are angles θ_1 and θ_2 changing as the player touches base?



a. The rate is $-\frac{14}{\sqrt{5}}$ ft/sec.

(Type an exact answer, using radicals as needed.)

b. When the player is 45′ from first base, what are the rates of change of the angles θ_1 and θ_2? $\frac{28}{225}, -\frac{28}{225}$ rad/sec

(Simplify your answers. Use a comma to separate answers as needed.)

c. As the player slides into second base at a rate of 15 ft / sec, what are the rates of change of the angles θ_1 and θ_2?

$\frac{1}{6}, -\frac{1}{6}$ rad/sec

(Simplify your answers. Use a comma to separate answers as needed.)

---

21. At what rate is the angle between a clock's minute and hour hands changing at 5 o'clock in the morning?

The angle is changing at $-\frac{11}{2}$ (1) ____________

(1)
- ○ deg • min.
- ● **deg / min.**
- ○ min / deg.
- ○ deg.