Derivatives

HW Question & Answer

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A body moves on a coordinate line such that it has a position $s(t) = t^2 - 5t + 4$ on the interval $0 \le t \le 6$ with s, position, in meters and t, time, in seconds.

a. Find the body's displacement and average velocity for the given time interval.

b. Find the body's speed and acceleration at the endpoints of the interval.

c. When, if ever, during the interval does the body change direction?

Answer:

a) Displacement: $s(6) - s(0) = 10\ m - 4\ m = 6\ m$

Average velocity $= \dfrac{displacement}{time} = \dfrac{6}{6} = 1\ m/s$

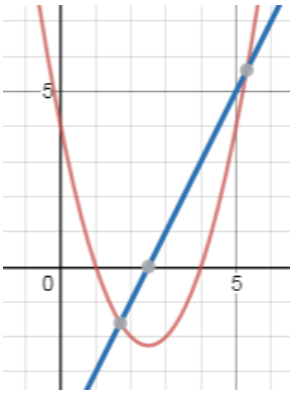
b) Velocity = derivative of position function. Speed is the absolute value of the velocity; therefore, $v = 2t - 5$.

Velocity at $t = 0$ is $v(0) = -5\ m/s$; the speed is $5\ m/s$. Velocity at the endpoint, $v(6) = 2(6) - 5 = 7\ m/s$, speed is also $7\ m/s$.

Acceleration is the derivative of the velocity function: $a(t) = 2$; that is, acceleration is constant and equal to $2\ m/s^2$.

c) Body changes direction when $v(t) = 0$ (while acceleration is not zero); in this case, since $v(t) = 2t - 5$ $v(t) = 0$ at $t = 2.5$ seconds.

A graph of the position (red) and velocity function (blue) created by desmos online calculator:



Notice: Position at $t = 0$ is 4; position at $t = 6$ is 10. Displacement, 6 units.

Initial velocity, -5; velocity $= 0$ at 2.5, (right in between 0 and 5); that is, the particle or body is moving to the left or backwards from 0 to 2.5 seconds; at 2.5 seconds changes direction and it moves to the right or forward.