

MAC2311 Calculus 1
Practice 2
Limits, Continuity.

Given the interval (a, b) on the x -axis with the point c inside, find the greatest value for $\delta > 0$ such that for all x , $0 < |x - c| < \delta \Rightarrow a < x < b$.

- 1) $a = -10, b = 0, c = -8$ 1) _____
 A) $\delta = 1$ B) $\delta = 2$ C) $\delta = 4$ D) $\delta = 8$

A function $f(x)$, a point c , the limit of $f(x)$ as x approaches c , and a positive number ϵ is given. Find a number $\delta > 0$ such that for all x , $0 < |x - c| < \delta \Rightarrow |f(x) - L| < \epsilon$.

- 2) $f(x) = 5x + 9, L = 29, c = 4$, and $\epsilon = 0.01$ 2) _____
 A) $\delta = 0.0025$ B) $\delta = 0.01$ C) $\delta = 0.002$ D) $\delta = 0.004$

- 3) $f(x) = 10x - 1, L = 29, c = 3$, and $\epsilon = 0.01$ 3) _____
 A) $\delta = 0.0005$ B) $\delta = 0.003333$ C) $\delta = 0.001$ D) $\delta = 0.002$

- 4) $f(x) = -7x + 10, L = -18, c = 4$, and $\epsilon = 0.01$ 4) _____
 A) $\delta = 0.001429$ B) $\delta = -0.0025$ C) $\delta = 0.005714$ D) $\delta = 0.002857$

- 5) $f(x) = -9x - 1, L = -19, c = 2$, and $\epsilon = 0.01$ 5) _____
 A) $\delta = -0.005$ B) $\delta = 0.000556$ C) $\delta = 0.001111$ D) $\delta = 0.002222$

- 6) $f(x) = \sqrt{x + 2}, L = 2, c = 2$, and $\epsilon = 1$ 6) _____
 A) $\delta = 9$ B) $\delta = 1$ C) $\delta = 3$ D) $\delta = 5$

- 7) $f(x) = \sqrt{7 - x}, L = 2, c = 3$, and $\epsilon = 1$ 7) _____
 A) $\delta = 6$ B) $\delta = 4$ C) $\delta = -5$ D) $\delta = 3$

Find the limit L for the given function f , the point c , and the positive number ϵ . Then find a number $\delta > 0$ such that, for all x , $0 < |x - c| < \delta \Rightarrow |f(x) - L| < \epsilon$.

- 8) $f(x) = 6x - 2, c = -3, \epsilon = 0.12$ 8) _____
 A) $L = -16; \delta = 0.02$ B) $L = 16; \delta = 0.03$
 C) $L = -20; \delta = 0.03$ D) $L = -20; \delta = 0.02$

- 9) $f(x) = \frac{x^2 + 2x + -80}{x + 10}, c = -10, \epsilon = 0.03$ 9) _____
 A) $L = 0; \delta = 0.03$ B) $L = -18; \delta = 0.03$
 C) $L = -16; \delta = 0.04$ D) $L = 2; \delta = 0.04$

- 10) $f(x) = \sqrt{8 - 2x}, c = -4, \epsilon = 0.5$ 10) _____
 A) $L = 4; \delta = 2.13$ B) $L = 4; \delta = 1.88$ C) $L = 5; \delta = 1.88$ D) $L = j-3; \delta = 0.88$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Prove the limit statement

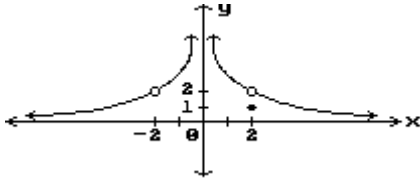
- 11) $\lim_{x \rightarrow 1} (2x - 3) = -1$ 11) _____

12) $\lim_{x \rightarrow 7} \frac{1}{x} = \frac{1}{7}$

12) _____

Find all points where the function is discontinuous.

13)



13) _____

- A) $x = -2, x = 0, x = 2$
- C) $x = 2$

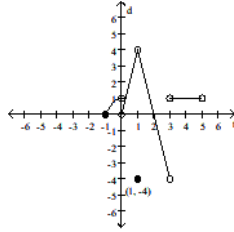
- B) $x = -2, x = 0$
- D) $x = 0, x = 2$

Answer the question.

14) Does $\lim_{x \rightarrow (-1)^+} f(x)$ exist?

14) _____

$$f(x) = \begin{cases} -x^2 + 1, & -1 \leq x < 0 \\ 4x, & 0 < x < 1 \\ -4, & x = 1 \\ -4x + 8, & 1 < x < 3 \\ 1, & 3 < x < 5 \end{cases}$$



A) Yes

B) No

Solve the problem.

15) To what new value should $f(1)$ be changed to remove the discontinuity?

15) _____

$$f(x) = \begin{cases} x^2 + 2, & x < 1 \\ 1, & x = 1 \\ x + 2, & x > 1 \end{cases}$$

A) 3

B) 1

C) 2

D) 4

16) To what new value should $f(2)$ be changed to remove the discontinuity?

16) _____

$$f(x) = \begin{cases} 2x - 4, & x < 2 \\ 2, & x = 2 \\ x - 2, & x > 2 \end{cases}$$

A) -8

B) -7

C) 0

D) -1

Find the intervals on which the function is continuous.

17) $y = \frac{2}{x+5} - 4x$

17) _____

- A) continuous everywhere
- C) discontinuous only when $x = -9$

- B) discontinuous only when $x = -5$
- D) discontinuous only when $x = 5$

$$18) y = \frac{3}{x^2 - 9}$$

18) _____

- A) discontinuous only when $x = -3$
 C) discontinuous only when $x = 9$

- B) discontinuous only when $x = -9$ or $x = 9$
 D) discontinuous only when $x = -3$ or $x = 3$

$$19) y = \sqrt{x^2 - 5}$$

19) _____

- A) continuous on the intervals $(-\infty, -\sqrt{5}]$ and $[\sqrt{5}, \infty)$
 B) continuous on the interval $[\sqrt{5}, \infty)$
 C) continuous everywhere
 D) continuous on the interval $[-\sqrt{5}, \sqrt{5}]$

Find the limit and determine if the function is continuous at the point being approached.

$$20) \lim_{x \rightarrow 4\pi} \sin(4x - \sin 4x)$$

20) _____

- A) does not exist; no
 C) 0; yes

- B) does not exist; yes
 D) 0; no

$$21) \lim_{x \rightarrow -\pi/2} \cos(5x - \cos 5x)$$

21) _____

- A) does not exist; yes
 C) 0; yes

- B) 0; no
 D) does not exist; no

$$22) \lim_{x \rightarrow 2\pi} \sin\left(\frac{-3\pi}{2} \cos(\tan x)\right)$$

22) _____

- A) does not exist; no
 C) does not exist; yes

- B) 1; no
 D) 1; yes

Find numbers a and b, or k, so that f is continuous at every point.

23)

23) _____

$$f(x) = \begin{cases} 8, & x < -4 \\ ax + b, & -4 \leq x \leq 4 \\ -24, & x > 4 \end{cases}$$

A) $a = -4, b = -40$

B) $a = -4, b = -8$

C) $a = 8, b = -24$

D) Impossible

24)

24) _____

$$f(x) = \begin{cases} x^2, & x < 3 \\ ax + b, & 3 \leq x \leq 5 \\ x + 20, & x > 5 \end{cases}$$

A) $a = 8, b = 15$

B) $a = -8, b = -15$

C) $a = 8, b = -15$

D) Impossible

25)

25) _____

$$f(x) = \begin{cases} 8x + 3, & \text{if } x < -1 \\ kx + 2, & \text{if } x \geq -1 \end{cases}$$

A) $k = -2$

B) $k = 7$

C) $k = 2$

D) $k = 9$

Answer Key

Testname: CALC1PRACTICE_2

- 1) B
- 2) C
- 3) C
- 4) A
- 5) C
- 6) C
- 7) D
- 8) D
- 9) B
- 10) B
- 11)

Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon/2$. Then $0 < |x - 1| < \delta$ implies that

$$\begin{aligned} |(2x - 3) + 1| &= |2x - 2| \\ &= |2(x - 1)| \\ &= 2|x - 1| < 2\delta = \varepsilon \end{aligned}$$

Thus, $0 < |x - 1| < \delta$ implies that $|(2x - 3) + 1| < \varepsilon$

- 12) Let $\varepsilon > 0$ be given. Choose $\delta = \min\{7/2, 49\varepsilon/2\}$. Then $0 < |x - 7| < \delta$ implies that

$$\begin{aligned} \left| \frac{1}{x} - \frac{1}{7} \right| &= \left| \frac{7 - x}{7x} \right| \\ &= \frac{1}{|x|} \cdot \frac{1}{7} \cdot |x - 7| \\ &< \frac{1}{7/2} \cdot \frac{1}{7} \cdot \frac{49\varepsilon}{2} = \varepsilon \end{aligned}$$

Thus, $0 < |x - 7| < \delta$ implies that $\left| \frac{1}{x} - \frac{1}{7} \right| < \varepsilon$

- 13) A
- 14) A
- 15) A
- 16) C
- 17) B
- 18) D
- 19) A
- 20) C
- 21) C
- 22) D
- 23) B
- 24) C
- 25) B