Marginal analysis

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NOTES

Marginal Cost, Revenue, and Profit

If x is the number of units of a product produced in some time interval, marginal cost, –or marginal revenue or marginal profit, is the instantaneous rate of change of cost -or revenue or profit - relative to production at a given production level.

We write:
$C(x)$: cost function for x items;
$C'(x)$: marginal cost: rate at which cost is changing when x items are being produced;
$R(x)$: Revenue; $R'(x)$: marginal revenue;
$P(x)$: total profit; $P'(x)$: marginal profit.
Recall that $P(x) = R(x) - C(x)$

Also:
Total cost of producing x items $= C(x)$;
Total cost of producing $(x+1)$ items $= C(x+1)$
Exact cost of producing the $(x+1)st$ item: $C(x+1) - C(x)$
And,
If $C(x)$ is the total cost of producing x items, the marginal cost function approximates the exact cost of producing the $(x+1)st$ item; therefore, $C'(x) \approx C(x+1) - C(x)$.

Example of cost analysis:

A company manufactures fuel tanks for automobiles. The total weekly cost (in dollars) of producing x tanks is given by $C(x) = 10,000 + 90x - 0.05x^2$:

a)Find the marginal cost function.
b)Find the marginal cost at a production level of 500 tanks per week and interpret the results.
c)Find the exact cost of producing the 501st item.

Solution:

a) $C'(x) = 90 - 0.1x$

b) Marginal cost: $C'(500) = 90 - 0.1(500) = \$40.00$

Interpretation of marginal cost: *At a production level of 500 tanks per week, the total production costs are increasing at the rate of $40 per tank.*

c) Total cost of producing 501 tanks per week:

$$C(501) = 10,000 + 90(501) - 0.05(501)^2 = \$42,539.95$$
$$C(500) = 10,000 + 90(500) - 0.05(500)^2 = \$42,500.00$$
$$C(501) - C(500) = 42,539.95 - 42,500.00 = \$39.95$$

Therefore, $39.95, is the exact cost of producing the 501st tank which is very close to $40.00 –marginal cost found in a).