ASYMPTOTES

Rational Functions

A *rational function* is a function f that is a quotient of two polynomials, that is,

$$f(x) = \frac{P(x)}{Q(x)}$$

where $P(x)$ and $Q(x)$ are polynomials with no common factor other than ± 1 and $Q(x)$ is not the zero polynomial. The domain of f consists of all inputs x for which $Q(x) \neq 0$.

Vertical Asymptotes (VA)

These asymptotes occur when the denominator $Q(x) = 0$. They have the form $x = a$.

Example: $f(x) = \frac{2x-1}{x^2-1} = \frac{2x-1}{(x-1)(x+1)}$

The denominator is 0 when $x = 1$ and $x = -1$. The vertical lines $x = 1$ and $x = -1$ are the vertical asymptotes.

Horizontal Asymptotes (HA)

These asymptotes occur when the degree of the numerator is less than or equal to the degree of the denominator.

- If the deg(num) < deg(den), then the horizontal asymptote is $y = 0$ (the x-axis)

Example: $f(x) = \frac{2x-1}{x^2-1}$ *deg(num) = 1 and deg(den) = 2*

So the horizontal asymptote is y = 0

- If the deg(num) = deg(den), then the asymptote is $y = \frac{a}{b}$, where a is the leading coefficient of the numerator and b is the leading coefficient of the denominator.

Example: $f(x) = \frac{2x^2-1}{x^2-1}$ *deg(num) = 2 and deg(den) = 2*

So the horizontal asymptote is y $= \frac{\mathbf{2}}{\mathbf{1}} = \mathbf{2}$

Slant or Oblique Asymptotes (SA)

These asymptotes occur when the degree of the numerator is one larger than the degree of the denominator. If this occurs you will have to do polynomial division and the quotient (the answer) is your slant asymptote.

Example: $f(x) = \frac{2x^2-1}{x-1}$ *deg(num) = 2 and deg(den) = 1*

$$\begin{array}{r|l} & 2x + 2 \\ \hline x-1 & 2x^2 - 1 \\ & \underline{2x^2 - 2x} \\ & \quad\quad 2x - 1 \\ & \quad\quad \underline{2x - 2} \\ & \quad\quad\quad\quad 1 \end{array}$$

The equation of the slant asymptote is $y = 2x + 2$

(Note the remainder does not affect this.)

NOTE: *You will never have both a SA and a HA on the same graph. Graphs can cross HA and SA, but will never cross VA.*