

Polar Coordinates and Vectors Sheet

MAC1114

url: imathesis.com

Polar Coordinates:

1. Multiple representations of points:

$$(r, \theta) = (r, \theta + 2n\pi) = (-r, \theta + \pi)$$

2. Relation between Polar Coordinates and Rectangular Coordinates:

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r^2 = x^2 + y^2$$

$$\tan \theta = y/x \quad \text{or} \quad \theta = \tan^{-1}(y/x)$$

Vectors:

For $\vec{v} = a\mathbf{i} + b\mathbf{j}$

1. Magnitude: $\|\vec{v}\| = \sqrt{a^2 + b^2}$

2. Unit vector in the direction of \vec{v} : $\frac{\vec{v}}{\|\vec{v}\|}$

3. Vector \vec{v} in terms of magnitude and direction:

$$\vec{v} = \|\vec{v}\| \cos \theta \mathbf{i} + \|\vec{v}\| \sin \theta \mathbf{j}$$

4. Dot product: given $\vec{u} = a_1\mathbf{i} + b_1\mathbf{j}$ and $\vec{v} = a_2\mathbf{i} + b_2\mathbf{j}$:

$$\vec{u} \cdot \vec{v} = a_1 \cdot a_2 + b_1 \cdot b_2$$

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos \theta.$$

Therefore,

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$$