

Appendix: calculation in R.

```
> View(Crime_v)
> attach(Crime_v)
> model2<-lm(V~po+hs+nhs+co+ba)
> shapiro.test(residuals(model2))
```

Shapiro-wilk normality test

```
data: residuals(model2)
W = 0.90029, p-value = 0.0004912
```

```
> bptest(model2)
```

studentized Breusch-Pagan test

```
data: model2
BP = 13.728, df = 5, p-value = 0.01744
```

```
> summary(model2)
```

Call:

```
lm(formula = V ~ po + hs + nhs + co + ba)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-906.69	-299.90	-87.02	179.90	1929.51

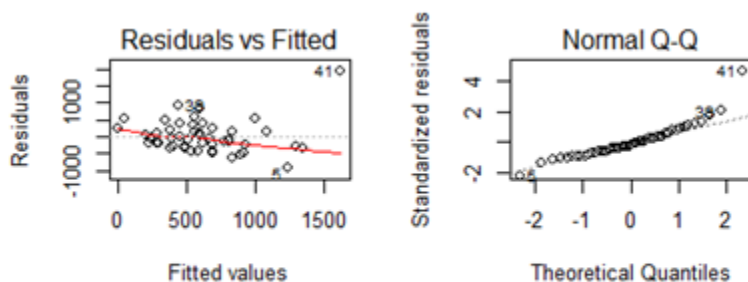
Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	1171.268	920.598	1.272	0.20995
po	21.010	5.998	3.503	0.00107 **
hs	-23.911	12.753	-1.875	0.06745 .
nhs	-7.097	19.593	-0.362	0.71893
co	-6.565	8.609	-0.763	0.44980
ba	26.273	26.805	0.980	0.33236

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 493.5 on 44 degrees of freedom
Multiple R-squared: 0.3356, Adjusted R-squared: 0.2601
F-statistic: 4.446 on 5 and 44 DF, p-value: 0.002303

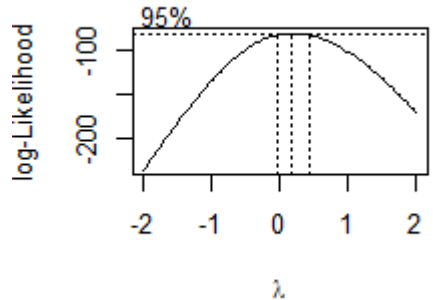
```
> par(mfrow=c(2,2))
> plot(model2)
```



```

> bc<-boxcox(model2)
> bc2<-boxcox(model2)
> lambda<-bc2$x[which.max(bc2$y)]
> lambda
[1] 0.1818182

```



```

> #lambda appr 1/5
> model2t<-lm(V^(1/5)~po+hs+nhs+co+ba)
> shapiro.test(residuals(model2t))

```

Shapiro-wilk normality test

```

data: residuals(model2t)
W = 0.97098, p-value = 0.2537

```

```

> bptest(model2t)

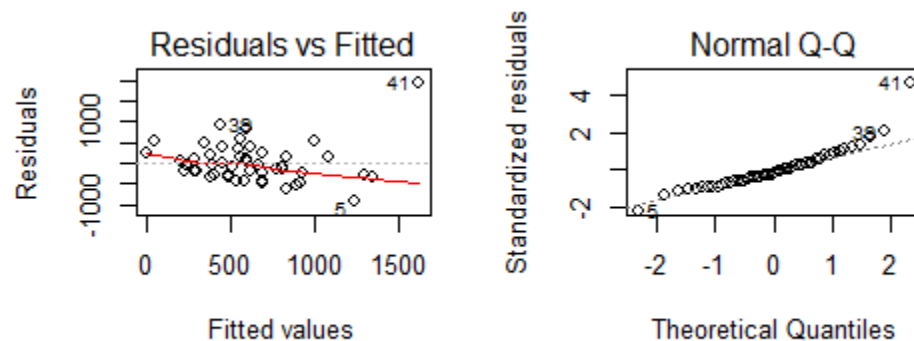
```

studentized Breusch-Pagan test

```

data: model2t
BP = 4.7318, df = 5, p-value = 0.4495

```



```

> summary(model2t)

```

```

Call:
lm(formula = V^(1/5) ~ po + hs + nhs + co + ba)

```

```

Residuals:
    Min       1Q   Median       3Q      Max
-0.84123 -0.39176  0.05075  0.34543  0.97243

```

```

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  4.9297569  0.9692996   5.086 7.26e-06 ***
po           0.0124581  0.0063156   1.973  0.05485 .

```

```

hs          -0.0423690  0.0134277  -3.155  0.00289 **
nhs         -0.0009855  0.0206299  -0.048  0.96211
co          -0.0168746  0.0090645  -1.862  0.06935 .
ba           0.0737617  0.0282232   2.614  0.01222 *

```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.5196 on 44 degrees of freedom
Multiple R-squared: 0.3695, Adjusted R-squared: 0.2979
F-statistic: 5.158 on 5 and 44 DF, p-value: 0.0008263

taking care of heteroskedasticity in model2 by WLS

```

> View(Crime_v)
> attach(Crime_v)
> model2<-lm(V~po+hs+nhs+co+ba)
> rs<-abs(residuals(model2))
> Crime_v$rs<-rs
> model2res<-lm(rs~po+hs+nhs+co+ba)
> w1<-fitted(model2res)
> Crime_v$w<-1/w1^2
> model2wls<-lm(V~po+hs+nhs+co+ba, weights = w)
> bptest(model2wls)

```

studentized Breusch-Pagan test

data: model2wls
BP = 13.728, df = 5, p-value = 0.01744

Education expenditure by States

```

> model10<-lm(exp~ipc+child)
> bptest(model10)

```

studentized Breusch-Pagan test

data: model10
BP = 8.9174, df = 2, p-value = 0.01158

```

> par(mfrow=c(2,2))
> plot(model10)
> shapiro.test(residuals(model10))

```

Shapiro-wilk normality test

data: residuals(model10)
W = 0.98224, p-value = 0.6494

```

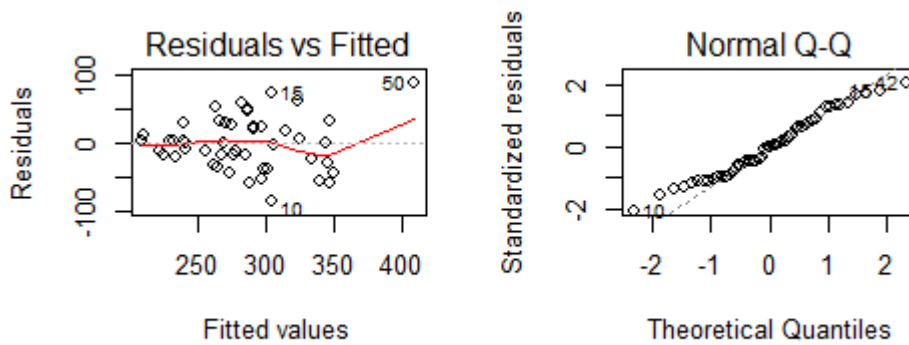
> education$res<-abs(residuals(model10))# saving abs value of residuals
> model10var<-lm(res~ipc+child)
> education$w<-fitted(model10var)
> model10WLS<-lm(exp~ipc+child, weights = 1/w^2)
> bptest(model10WLS)

```

studentized Breusch-Pagan test

data: model10WLS
BP = 8.9174, df = 2, p-value = 0.01158

```
> plot(model10WLS)
```

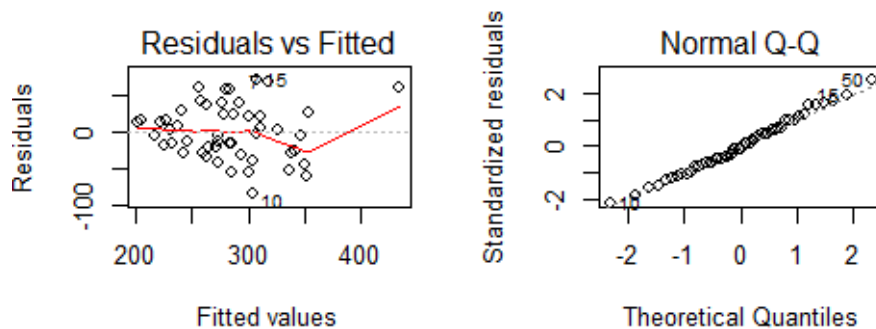


```
> model11WLS<-lm(exp~ipc+child, weights = 1/urbe)
> bptest(model11WLS)
```

studentized Breusch-Pagan test

```
data: model11WLS
BP = 8.9174, df = 2, p-value = 0.01158
```

```
> plot(model11WLS)
```



Box Cox transformation

```
> attach(education)
> model10<-lm(exp~ipc+child)
> bc<-boxcox(model10)
> shapiro.test(residuals(model10))
```

Shapiro-wilk normality test

```
data: residuals(model10)
W = 0.98224, p-value = 0.6494
```

```
> bptest(model10)
```

studentized Breusch-Pagan test

```
data: model10
BP = 8.9174, df = 2, p-value = 0.01158
```

```
> lambda<-bc2$x[which.max(bc2$y)]
> lambda
[1] -0.5000
```

```
> model10B<-lm(exp^-0.5~ipc+child)
> shapiro.test(residuals((model10B)))
```

Shapiro-wilk normality test

```
data: residuals((model10B))
W = 0.97639, p-value = 0.4115
```

```
> bptest(model10B)
```

studentized Breusch-Pagan test

```
data: model10B
BP = 2.9424, df = 2, p-value = 0.2297
```

Same model in Minitab 18:

Regression Analysis: exp versus ipc, child

Box-Cox transformation

Rounded λ -0.5
 Estimated λ -0.726751
 95% CI for λ (*, 0.482749)

Model Summary for Transformed Response

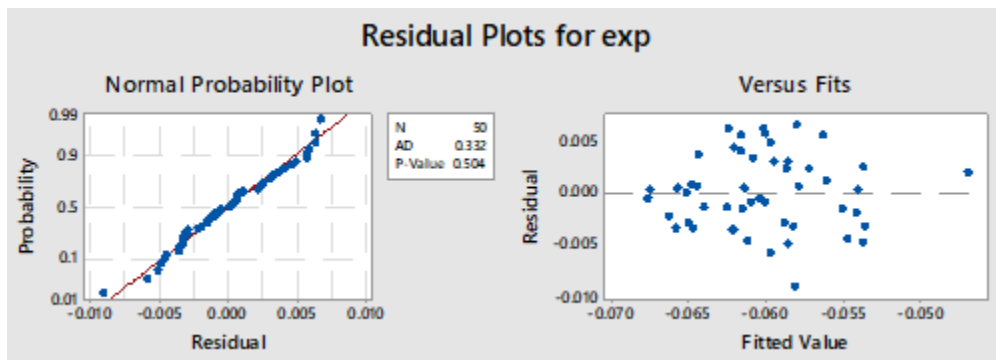
S	R-sq	R-sq(adj)	R-sq(pred)
0.0037444	57.44%	55.63%	52.67%

Coefficients for Transformed Response

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-0.1294	0.0113	-11.45	0.000	
ipc	0.000007	0.000001	7.76	0.000	1.10
child	0.000115	0.000029	4.00	0.000	1.10

Regression Equation

$-\exp^{-0.5} = -0.1294 + 0.000007 \text{ ipc} + 0.000115 \text{ child}$



```
> View(Crime_d)
> attach(Crime_d)
> model1<-lm(x2~x3+x4+x5+x6+x7)
> library(lmtest)
> library(MASS)
> bptest(model1)
```

studentized Breusch-Pagan test

```
data: model1
BP = 13.728, df = 5, p-value = 0.01744
```

```
> shapiro.test(residuals(model1))
```

Shapiro-wilk normality test

```
data: residuals(model1)
W = 0.90029, p-value = 0.0004912
```

```
> Crime_d$res<-abs(residuals(model1))
> modelVar<-lm(res~x3+x4+x5+x6+x7)
> Crime_d$w<-predict(modelVar)
> model1WLS<-lm(x2~x3+x4+x5+x6+x7, weights = 1/w^2)
> bptest(model1WLS)
```

studentized Breusch-Pagan test

```
data: model1WLS
BP = 13.728, df = 5, p-value = 0.01744
```

```
> View(hmc)
> attach(hmc)
> modelOLS<-lm(achieve~self+ability)
> bptest(modelOLS)
```

studentized Breusch-Pagan test

```
data: modelOLS
BP = 7.3664, df = 2, p-value = 0.02514
```

```
> shapiro.test(residuals(modelOLS))
```

Shapiro-wilk normality test

```
data: residuals(modelOLS)
W = 0.97271, p-value = 0.6157
```

```
> hmc$res<-abs(residuals(modelOLS))
> modelSD<-lm(res~self+ability)
> hmc$w<-predict(modelSD)
> modelWLS<-lm(achieve~self+ability, weights = 1/w^2)
> bptest(modelWLS)
```

studentized Breusch-Pagan test

```
data: modelWLS
BP = 7.3664, df = 2, p-value = 0.02514
```

```
> View(education)
> attach(education)
> model3<-lm(Exp~inpc+u18)
> bptest(model3)
```

studentized Breusch-Pagan test

```
data: model3
BP = 8.9174, df = 2, p-value = 0.01158
```

```
> shapiro.test(residuals(model3))
```

```
Shapiro-wilk normality test
```

```
data: residuals(model3)
W = 0.98224, p-value = 0.6494
```

```
> model3c<-lm(Exp~inpc+u18, subset= -50)
> bptest(model3c)
```

```
studentized Breusch-Pagan test
```

```
data: model3b
BP = 5.0421, df = 2, p-value = 0.08038
```

```
> model3m<-lm(1/Exp~inpc+u18)
> bptest(model3m)
```

```
studentized Breusch-Pagan test
```

```
data: model3b
BP = 0.66446, df = 2, p-value = 0.7173
```

```
plot(fitted(model5), residuals(model5), main="residual plot model5")
```

```
> par(mfrow=c(2,2))
> plot(model7)
> alpha$r<-abs(residuals(model7))
> model7sd<-lm(r~pluto, data=alpha)
> alpha$w<-fitted(model7)
> model7WLS<-lm(alpha~pluto, weights = 1/w^2, data=alpha)
```

```
model2<-lm(Price~Ah+A1ot, homep)
```

```
> shapiro.test(residuals(model2))
```

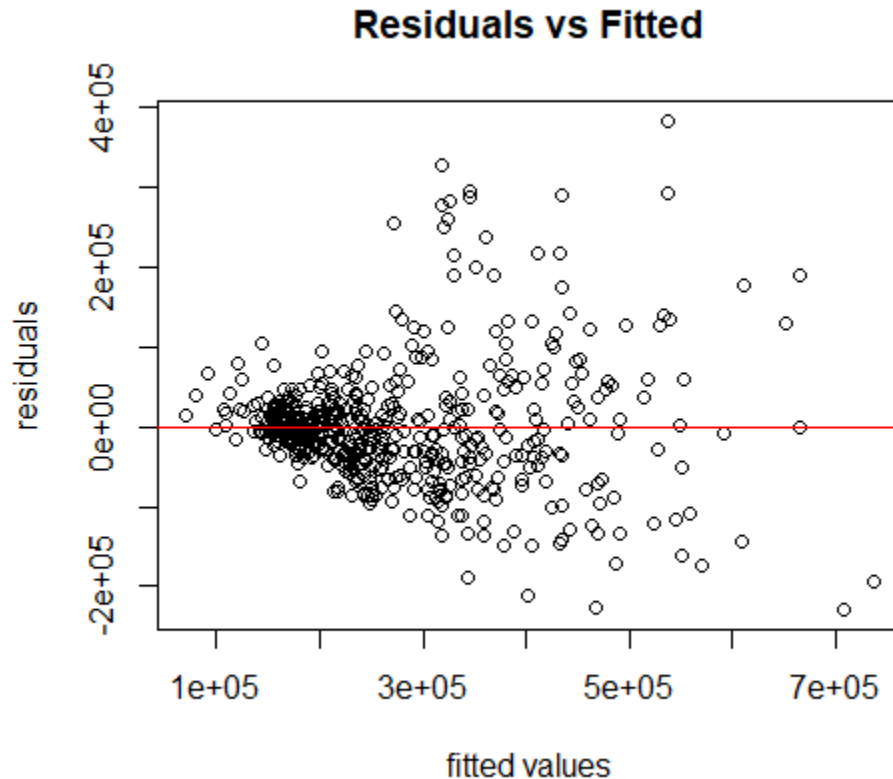
```
Shapiro-wilk normality test
```

```
data: residuals(model2)
W = 0.90829, p-value < 2.2e-16
```

```
> bptest(model2)
```

```
studentized Breusch-Pagan test
```

```
data: model2
BP = 85.21, df = 2, p-value < 2.2e-16
plot(fitted(model2), residuals(model2), xlab="fitted values", ylab="residuals", mai
n="Residuals vs Fitted")
```



```
> summary(model2)
```

Call:

```
lm(formula = Price ~ Ah + Alot, data = homep)
```

Residuals:

Min	1Q	Median	3Q	Max
-228421	-38178	-5506	25494	383423

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	-1.027e+05	1.265e+04	-8.121	3.39e-15	***
Ah	1.560e+02	4.871e+00	32.019	< 2e-16	***
Alot	1.151e+00	2.964e-01	3.882	0.000117	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 78070 on 519 degrees of freedom

Multiple R-squared: 0.6808, Adjusted R-squared: 0.6796

F-statistic: 553.5 on 2 and 519 DF, p-value: < 2.2e-16

```
> model3<-lm(log(Price)~log(Ah)+log(Alot), homep)
```

```
> bptest(model3)
```

studentized Breusch-Pagan test

data: model3

BP = 34.293, df = 2, p-value = 3.575e-08

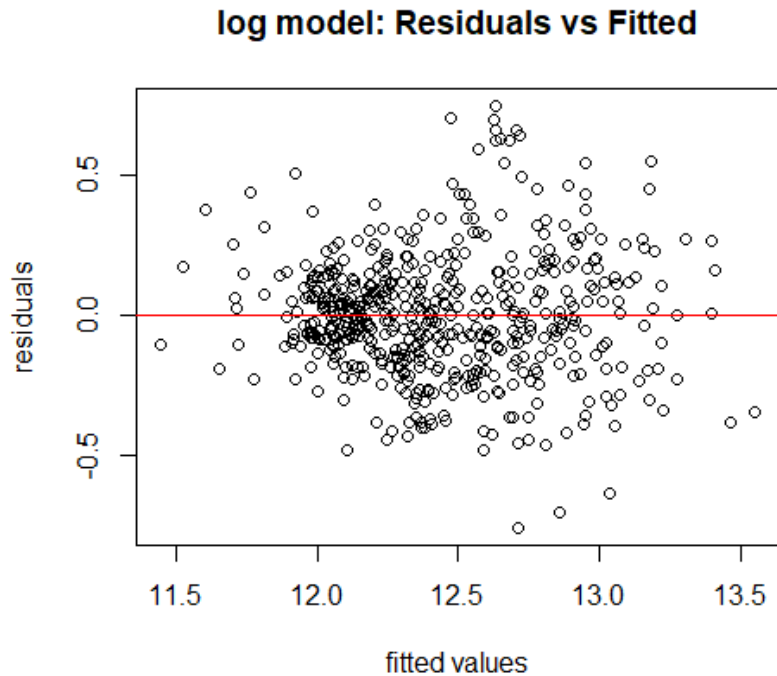
```
> plot(fitted(model3), residuals(model3), xlab="fitted values", ylab="residuals", main="log model: Residuals vs Fitted")
```



```

> shapiro.test(residuals(model3))
> vif(model2)
      Ah      A1ot
1.025445 1.025445

```



```

> summary(model2)

```

Call:

```
lm(formula = Price ~ Ah + A1ot, data = homep)
```

Residuals:

Min	1Q	Median	3Q	Max
-228421	-38178	-5506	25494	383423

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	-1.027e+05	1.265e+04	-8.121	3.39e-15	***
Ah	1.560e+02	4.871e+00	32.019	< 2e-16	***
A1ot	1.151e+00	2.964e-01	3.882	0.000117	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 78070 on 519 degrees of freedom
Multiple R-squared: 0.6808, Adjusted R-squared: 0.6796
F-statistic: 553.5 on 2 and 519 DF, p-value: < 2.2e-16

```

> res<-(residuals(model2))^2
> homep$r<-log(res)
> model2r<-lm(r~Ah+A1ot)
> homep$w<-exp(fitted(model2r))
> model2wls<-lm(Price~Ah+A1ot, weights = 1/w)
> bptest(model2wls)

```

studentized Breusch-Pagan test

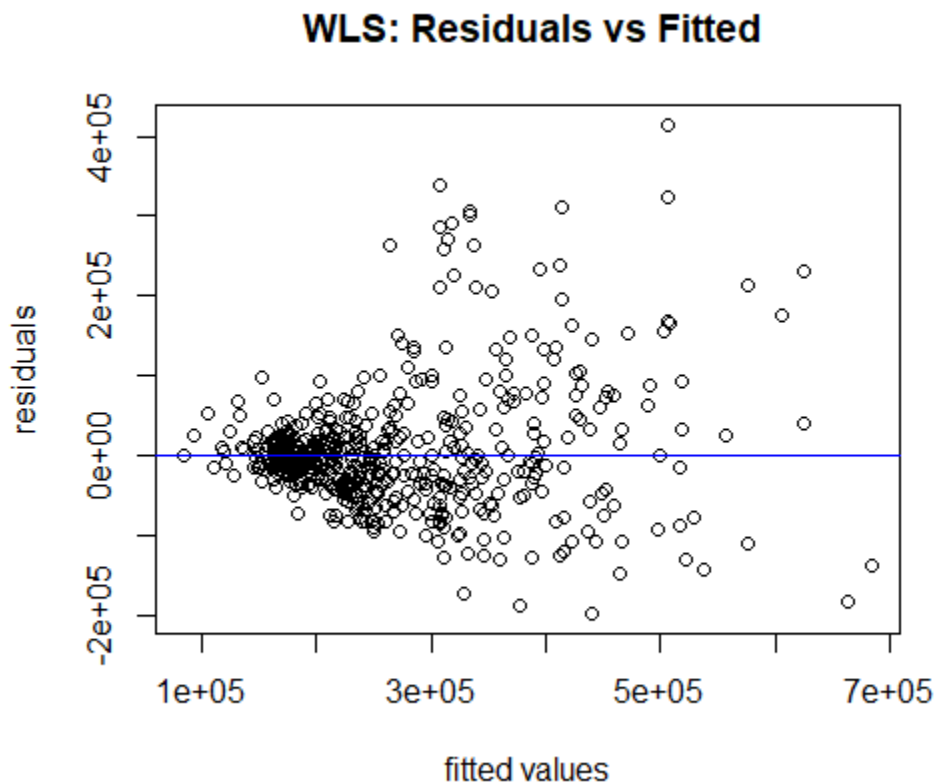
```
data: model2wls  
BP = 85.21, df = 2, p-value < 2.2e-16  
> summary(model2wls)
```

```
Call:  
lm(formula = Price ~ Ah + Alot, weights = 1/w)
```

```
Weighted Residuals:  
      Min       1Q   Median       3Q      Max  
-4.6268 -1.1948 -0.2279  1.0238  9.2133
```

```
Coefficients:  
              Estimate Std. Error t value Pr(>|t|)  
(Intercept) -6.934e+04  9.642e+03  -7.191 2.26e-12 ***  
Ah           1.419e+02  5.089e+00  27.886 < 2e-16 ***  
Alot         8.548e-01  2.608e-01   3.278  0.00112 **  
---  
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 2.087 on 519 degrees of freedom  
Multiple R-squared:  0.6272, Adjusted R-squared:  0.6257  
F-statistic: 436.5 on 2 and 519 DF, p-value: < 2.2e-16
```



```

> bc<-boxcox(model2)
> lambda<-bc$x[which.max(bc$y)]
> lambda
[1] -0.2626263
> model2bx<-lm(Price^(-0.3)~Ah+A1ot, homep)
> shapiro.test(residuals(model2bx))

```

Shapiro-wilk normality test

```

data: residuals(model2bx)
W = 0.98978, p-value = 0.00108
> bptest(model2bx)

```

studentized Breusch-Pagan test

```

data: model2bx
BP = 31.695, df = 2, p-value = 1.311e-07

```

```

Call:
lm(formula = log(Price) ~ log(Ah) + log(A1ot), data = homep)

```

```

Residuals:
    Min       1Q   Median       3Q      Max
-0.75717 -0.13657 -0.00747  0.12505  0.74716

```

```

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.96361    0.31286   6.276 7.33e-10 ***
log(Ah)      1.21976    0.03401  35.867 < 2e-16 ***
log(A1ot)    0.11034    0.02412   4.575 5.97e-06 ***
---

```

```

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

```

Residual standard error: 0.2216 on 519 degrees of freedom
Multiple R-squared:  0.7374, Adjusted R-squared:  0.7364
F-statistic: 728.8 on 2 and 519 DF, p-value: < 2.2e-16

```

```

> library(lmtest)
> coeftest(model3, vcov=hccm)

```

t test of coefficients:

```

            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.963614    0.351710   5.5831 3.815e-08 ***
log(Ah)      1.219758    0.033665  36.2327 < 2.2e-16 ***
log(A1ot)    0.110340    0.029938   3.6856 0.0002522 ***
---

```

```

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

```

> model10<-lm(Size~Age)
> plot(fitted(model10), residuals(model10), xlab="fitted values", ylab="residuals",
main="Residuals vs Fitted")
> shapiro.test(residuals(model10))

```

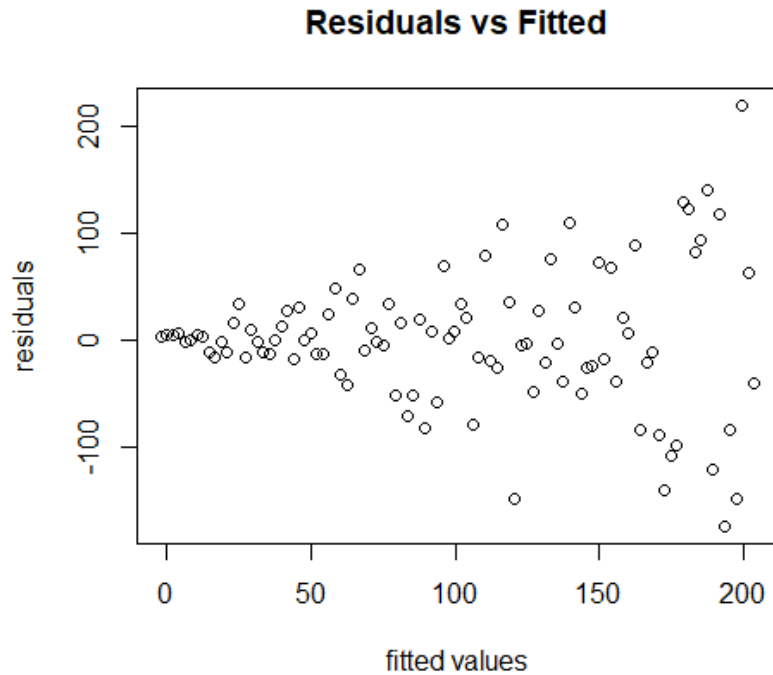
Shapiro-wilk normality test

```
data: residuals(model10)
w = 0.96429, p-value = 0.008227
```

```
> bptest(model10)
```

studentized Breusch-Pagan test

```
data: model10
BP = 31.152, df = 1, p-value = 2.386e-08
```



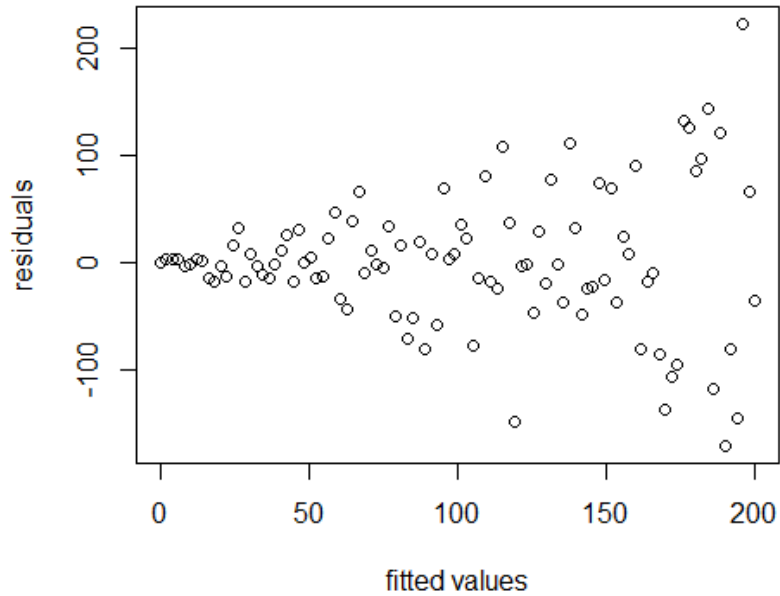
```
> res<-(residuals(model10))^2
> age_size$r<-log(res)
> model10r<-lm(r~Age, data=age_size)
> age_size$w<-exp(fitted(model10r))
> model10wls<-lm(Size~Age, weights = 1/w, data=age_size)
> bptest(model10wls)
```

studentized Breusch-Pagan test

```
data: model10wls
BP = 31.152, df = 1, p-value = 2.386e-08
```

```
> plot(fitted(model10wls), residuals(model10wls), xlab="fitted values", ylab="residuals", main="WLS: Residuals vs Fitted")
```

WLS: Residuals vs Fitted



Prof Crowson:

```
> age_size$rs<-abs(rstandard(model20))  
> model20aux<-lm(rs~Age, data=age_size)  
> un<-fitted(model20aux)  
> age_size$w<-((un-mean(un))/sd(un))^2  
> model20wls<-lm(Size~Age, weights = 1/w, data=age_size)  
> plot(fitted(model20wls),residuals(model20wls), main="WLS st residuals vs fitted")
```

WLS st residuals vs fitted

