

A collection of objects whose contents can be clearly determined.

Elements or **members** are the objects in a set.

A set must be **well-defined**, meaning that its contents can be clearly determined.

The order in which the elements of the set are listed is not important.

Representing a Set Using the Roster Method

$$\mathbf{N} = \{ 1, 2, 3, 4, 5 \dots \}$$

Set-Builder Notation

Set A is the set of natural numbers less than 5. $A = \{1,2,3,4\}$

$\mathbf{N} = \{ x \mid x \in \mathbf{N} \}$ use of \in : ... is an element of; \notin ...is not an element of.

$$7 \notin \{1,2,3,4,5\}$$

$\{a\} \in \{a, b\}$ False: $\{a\}$ is a set and the set $\{a\}$ is not an element of the set $\{a, b\}$.

Empty set: \emptyset

Inequality Notation and Sets

Inequality Symbol and Meaning	Set-Builder Notation	Example Roster Method
$x < a$ x is less than a.	$\{x \mid x \in \mathbf{N} \text{ and } x < 4\}$ x is a natural number less than 4.	$\{1, 2, 3\}$
$x \leq a$ x is less than or equal to a.	$\{x \mid x \in \mathbf{N} \text{ and } x \leq 4\}$ x is a natural number less than or equal to 4.	$\{1, 2, 3, 4\}$
$x > a$ x is greater than a.	$\{x \mid x \in \mathbf{N} \text{ and } x > 4\}$ x is a natural number greater than 4.	$\{5, 6, 7, 8, \dots\}$
$x \geq a$ x is greater than or equal to a.	$\{x \mid x \in \mathbf{N} \text{ and } x \geq 4\}$ x is a natural number greater than or equal to 4.	$\{4, 5, 6, 7, \dots\}$

Express each of the following sets using the roster method:

a. $\{x \mid x \in \mathbf{N} \text{ and } x \leq 100\}$

Solution: $\{1, 2, 3, 4, \dots, 100\}$

Cardinality and Equivalent Sets: $n(A)$ is the number of distinct elements in set A .

Two sets are equivalent if they have the same cardinality.

Equal sets: they contain exactly the same elements.

Finite set: its cardinality is a Natural number.

Subsets: \subseteq

A is a subset of B : $A \subseteq B$

The notation $A \not\subseteq B$ means that A is not a subset of B

Every set is a subset of itself.

Proper Subsets: \subset

Set A is a proper subset of set B , expressed as $A \subset B$, if set A is a subset of set B and set A and B are not equal.

Subsets and the Empty Set: the empty set is a subset of every set A . $\emptyset \subset A$

The number of subsets of a set with n elements is 2^n .

The number of proper subsets of a set with n elements is $2^n - 1$.

The Number of Subsets of Infinite Sets

There are \aleph_0 natural numbers.

It has 2^{\aleph_0} subsets.

It has $2^{\aleph_0} - 1$ proper subsets

$2^{\aleph_0} > \aleph_0$

Denote 2^{\aleph_0} by \aleph_1

$\aleph_1 > \aleph_0$

\aleph_0 is the "smallest" transfinite cardinal number in an infinite hierarchy of different infinities.

Universal Sets and Venn Diagrams:

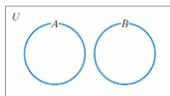
The universal set is a general set that contains all elements under discussion.

Venn diagrams show the visual relationship among sets.

Universal set is represented by a rectangle.

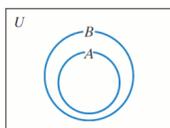
Subsets within the universal set are depicted by circles, or sometimes ovals or other shapes.

Disjoint Sets: Two sets that have no elements in common.

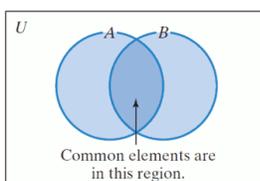


Proper Subsets:

All elements of set A are elements of set B .



If set A and set B have at least one element in common, then the circles representing the sets must overlap.



The Complement of a Set: it is symbolized A' is the set of all elements in the universal set that are not in A .

The Intersection of Sets: $A \cap B$: it is the set of elements common to both A and B .

The Union of Sets: $A \cup B$: Elements that are members of set A or set B or both.

Given: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 7, 9\}$, $B = \{3, 7, 8, 10\}$

Find $(A \cup B)'$

Solution: $A \cup B = \{1, 3, 7, 8, 9, 10\}$; $(A \cup B)' = \{2, 4, 5, 6\}$

The Cardinal Number of the Union of Two Finite Sets:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Example: Set Operations with Three Sets

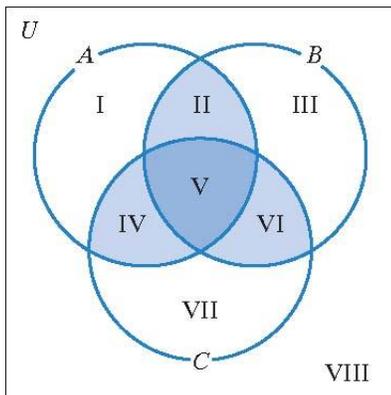
Given: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$; $A = \{1, 2, 3, 4, 5\}$; $B = \{1, 2, 3, 6, 8\}$; $C = \{2, 3, 4, 6, 7\}$

Find $A \cap (B \cup C)$:

Step 1: Find the complement of C . Ans: $C' = \{1, 5, 8, 9\}$

Step 2: Find the union of B and C' . Ans: $B \cup C' = \{1, 2, 3, 5, 6, 8, 9\}$

Step 3: Find the intersection. $A \cap (B \cup C') = \{1, 2, 3, 5\}$



The Region Shown in Dark Blue

Region V This region represents elements that are common to sets A , B , and C : $A \cap B \cap C$.

The Regions Shown in Light Blue

Region II This region represents elements in both sets A and B that are not in set C : $(A \cap B) \cap C'$.

Region IV This region represents elements in both sets A and C that are not in set B : $(A \cap C) \cap B'$.

Region VI This region represents elements in both sets B and C that are not in set A : $(B \cap C) \cap A'$.

The Regions Shown in White

Region I This region represents elements in set A that are in neither sets B nor C : $A \cap (B' \cap C')$.

Region III This region represents elements in set B that are in neither sets A nor C : $B \cap (A' \cap C')$.

Region VII This region represents elements in set C that are in neither sets A nor B : $C \cap (A' \cap B')$.

Region VIII This region represents elements in the universal set U that are not in sets A , B , or C : $A' \cap B' \cap C'$.

De Morgan's Laws

1. The complement of the intersection of two sets is the union of the complements of those sets.
 $(A \cap B)' = A' \cup B'$
2. The complement of the union of two sets is the intersection of the complements of those sets.
 $(A \cup B)' = A' \cap B'$

Survey Problems: Start with the intersection of the sets, the innermost region, and work outward.

Example 1: Surveying People's Attitudes

A survey is taken that asks 2000 randomly selected U.S. and Mexican adults the following question: Do you agree or disagree that the primary cause of poverty is societal injustice?

The results of the survey showed that:

1060 people agreed with the statement

400 Americans agreed with the statement.

If half the adults surveyed were Americans

- a. How many Mexicans agreed with the statement?
- b. How many Mexicans disagreed with the statement?

Example 2: Sixty people were contacted and responded to a movie survey. The following information was obtained:

- a. 6 people liked comedies, dramas, and science fiction.
- b. 13 people liked comedies and dramas.
- c. 10 people liked comedies and science fiction.
- d. 11 people liked dramas and science fiction.
- e. 26 people liked comedies.
- f. 21 people liked dramas.
- g. 25 people liked science fiction.

Use a Venn diagram to illustrate the survey's results.

How many of those surveyed liked:

- a. comedies, but neither dramas nor science fiction?
- b. dramas and science fiction, but not comedies?
- c. dramas or science fiction, but not comedies?
- d. exactly one movie style?
- e. at least two movie styles?
- f. none of the movie styles?