The binomial distribution

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The binomial experiment:

Properties of the binomial distribution:

The binomial distribution for a discrete variable X must satisfy the following conditions:

- The number of observations trials in the experiment– is fixed in advanced.
- The observations or trails are independent: the outcome of one does not influence the outcome of any other trials. It also means that trials are identical.
- Each observation represents one of two outcomes: success or failure.
- The probability of success p is constant for each outcome.

A binomial experiment:

Suppose we toss a die five times. What is the probability of getting exactly 2 fours?
One possible outcome would be getting two fours in a row and then three no four numbers like 1, 2,3, 5 and 6. So we would have 44NNN. Notice, however, that there are ${}_5C_2$ ways of arranging 2 successes and 3 failures in a five trials experiment. In each of the particular outcomes the probability is given by: $\left(\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\right)$; that is $\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$. Combining in one formula the number of possible arrangements and the probability of each of the outcomes, we have:
$p_{(2)} = \binom{5}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 =$

Probability mass function:

Considering that,

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

Then, the probability mass function for the binomial distribution is given by:

$$p(x) = \binom{n}{x} p^x \cdot q^{n-x}$$

Where n represents the number of trials; x, the number of successes; p, the probability of success and q the probability of failure. Since outcome of a each trial will be a success of failure, both, p and q add up to the total probability of 1. Therefore, $q = 1 - p$.

The binomial expansion...

The binomial theorem,

$$(a+b)^n = \sum_{j=0}^{n} \binom{n}{j} a^{n-j} b^j$$

Can be expressed in terms of p (probability of success) and q(probability of failure), as follows:

$$(p+p)^n = \sum_{x=0}^{n} \binom{n}{x} q^{n-x} p^x =$$

$$\binom{n}{x} q^n p^0 + \binom{n}{x} q^{n-1} p^1 + \binom{n}{x} q^{n-2} p^2 + \binom{n}{x} q^{n-3} p^3 + \cdots$$